Empirical Evaluation of Intervals and Fretting Systems in Persian Art Music

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Tār and *sitār*, which are sister fretted long-necked plucked string instruments (Figure 1), are among the main and most recognized instruments in Persian art music. In addition, most vocalists of Persian art music take lessons in *sitār* (often transliterated[1] as *setār*) in order to learn the *radīf* (ordered repertoire) and practice the intervals and the ornamentations. Both these instruments use an X-tone fretting system, with values of X between 15 and 24 (although mostly 17 or 18). This translates to some musical tones, typically referred to as neutral (or median) tones, that lie in between (some of) the tones of the 12-tone western musical scale. The problem is that there is no nationally recognized or standardized fretting system for the Iranian string instruments and, in general, there is no standardized parent scale for Persian art music.

[2] The fretting (*dastān-nishānī*)[2] of string instruments, particularly with respect to the neutral tones, has remained a relatively subjective tradition in Iran.[3] In fact, the variability or flexibility of the neutral interval sizes has been considered a characteristic of Persian art music by some musicologists (Farhat 1990, 16) (Talai 2017, 22). Although such variability would not be an issue in solo performances, it can create a challenge and a debate at every ensemble performance. The choice of the neutral interval sizes is usually made subjectively

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1. This article adopts the transliteration guideline of the International Journal of Middle East Studies (IJMES) for technical terms (requiring diacritics) and personal names (not requiring diacritics), but not completely for the titles of books or articles (where diacritics were fully added in this article for clarity, although not required by IJMES guideline). The name of the Muslim scholars from the Greater Iran are presented as they would appear in contemporary Persian texts (rather than the Arabicized form with the article *al* as in al-Urmawi or al-Farabi).
2. Fret, or particularly *dastān*, in this context can include markings on the neck of the instrument, as stated by Urmawi (2001, 9), Shirazi (2008, 115) and Maraghi (1966, 14), and does not necessarily mean a physical fret (*pardih*) as placed on a *tār*/*sitār*.
3. Talai (2017, 22) states that the interval sizes in Persian art music (specifically the seconds) can vary according to the genus/mode type, the school of music (*maktab*) and the personal style.

Figure 1. Samples of the Iranian instruments *tār* (left) and *sitār* (right).

(and typically by the most senior members) in every ensemble performance. If one or more standardized scales with interval sizes close to common practice can be proposed, that theoretical contribution will potentially have value to performers as well as musicologists.

[3] To this end, an extensive historical literature survey was conducted on parent scales and fretting systems in Persian art music and the results, along with some theoretical examinations and proposals, were published (Poorhaydari 2022). That study was followed by an empirical evaluation of the intervals and fretting systems along with amended theoretical examinations based on the modification of Greek tuning systems, which are presented here. Prior to presenting the methodology, the results and the proposed parent scales, a short and selective review of the first study along with some background information on Persian art music as it pertains to the subject of this article may be of interest to the readers.

BACKGROUND INFORMATION

[4] In the early Islamic era, Abu Nasr Farabi (872–950) presented specific theoretical models (ratios and scales) for the fretting and tuning of various instruments, including *ṭanbūr khurāsānī* and oud (Farabi 1996). Approximately three centuries later, Safi al-Din Urmawi (1216–1294) presented a 17-tone scale on a monochord that was implicitly considered the unified theoretical model for all instruments and musical cycles (modes). The scale was slightly modified (and completed in my opinion) by ʿAbd al-Qadir Ghaybi Maraghi (d. 1435) in the fifteenth century. The main issue with Urmawi-Maraghi's scale was the direct use of Pythagorean sub-tone intervals in the construction of the whole tone that resulted in neutral intervals not common in practice (Wright 1978, 32). To be more specific, the Pythagorean limma-comma construction of a whole tone (i.e. LLC, LCL or, rarely, CLL, where L stands for a limma of 256:243 or ca. 90.2 c and C for a comma of 531,441:524,288 or ca. 23.46 c) that was advocated by the Systematists[4] produces neutral tones with interval sizes of L + C (an apotome of ca. 113.7 c) or 2L (ca. 180.4 c) above their main lower tones. Even the Systematists themselves admitted that most practitioners used neutral tones midway between some lower and higher tones in the scale (Shirazi 2008, 116), e.g., between the Pythagorean *zāʾid* (90.2 c) and *sabbābih* (203.9 c) that yields a neutral second of 4608:4235 (146.1 c) through an arithmetic mean calculation.

[5] The discrepancy between the theoretical and practical fretting systems and the dispute over the size of the neutral intervals in Persian art music continued to the modern time. After Vaziri (1934, Part 2, 4) proposed the 24-tone equal temperament (ET) in the early twentieth century (resulting in a neutral second of exactly 150 c), several musicologists objected to the theoretical proposal and attempted to determine the intervals empirically using acoustic measurements (Barkeshli (1976, 87), Farhat (1990, 16), Sepanta (1998, 68) and During (1985, 79–118)) or some other methods (Kiani (1992, 182) and Beizai (2003, 67)). Their proposed neutral second (*mujannab*) interval sizes are summarized in Table 1. As it can be seen, there is a large variation among the proposed interval sizes, ranging from 120 c to

4. The term "Systematists" refers to the music scholars of the Islamic Golden Age, specifically those between the 13th and the 15th centuries, who systematized the scale and some other aspects of the music theory (Farmer 1929, 206).

Musicologist	Approximate Time of Investigation	Method of Investigation	Interval Size, c	Provided Ratio
Ali Naqi Vaziri	1920's	Equal Temperament	150	-
Mehdi Barkeshli	1940's	Acoustic Measurement	120, 180	-
Hormoz Farhat	1950's	Acoustic Measurement	135, 160	-
Sasan Sepanta	1960's	Acoustic Measurement	135	-
Jean During	1970's	Acoustic Measurement	145[5]	162:149
Majid Kiani	1980's	Pictorial Measurement	144	88:81
Dariush Talai	1990's	Personal Experience	120, 140, 180 (150 tempered)	-
Siavash Beizai	2000's	Theoretical Proposal	143, 151	88:81, 12:11

Table 1. Neutral second (*mujannab*) interval size according to the twentieth-century musicologists.

180 c. Nevertheless, it is evident that toward the end of the twentieth century, the proposed values converged to a narrower range of ca. 140–150 c and even a tempered size of 150 c was considered (although not necessarily favored then) by Talai (1993, 19), i.e., a return to the earlier proposal by Vaziri.

[6] It should be noted the variability of the neutral intervals and the large range of the values measured by the twentieth-century musicologist are inherent to Persian art music. Persian art music repertoires (*radīf*s) are ordered collections of melodies or "melody models" (Farhat 1990, 21), each called a *gūshih*, that are gathered for pedagogical purposes by some musicians in the nineteenth century Iran and divided into several modal groupings, referred to as *dastgāh*s (systems). They represent what was inherited and could be gathered in nineteenth-century Iran. Since the *gūshih*s originate from different sources, locations and traditions and

5. During did not explicitly provide an average value of 145 c for *mujannab*. This value is the average of all measurements reported by During (2006, 330-332) in different *dastgāhs* and from several musicians), which coincides exactly with Farabi's *mujannab fārs* of 162:149 (144.8 c).

they have been transferred aurally by musicians (Talai 1993, 7–10), it is not possible (or meaningful) to determine the exact intervals within each mode (and in each *radīf*) or how they evolved over time. It is understandable that the neutral intervals may be of different sizes in different received *gūshih*s and therefore regarded as “flexible”. But this does not mean that they ought to remain so, as suggested in this article by proposing some standardized scales.

METHODOLOGY AND INITIAL TESTS

[7] In order to examine the fretting systems and measure the intervals statistically, two empirical methods, namely digital measurement through an online survey and pictorial calculation, were adopted. The online survey was conducted among a number of musicians (mostly *tār/sitar* players, but also including some *santūr*, Persian hammered dulcimer, players) of a range of experience with two goals. First, to gather their opinions on the variability and subjectivity of the current fretting systems and the need for one or more standardized fretting systems. Second, to determine the size of selected key intervals as played by the musicians. The pictorial measurement was conducted as a verification method for the selected intervals as well as to measure the rest of intervals in the parent scale. The pictorial method has the potential for examining all intervals from the pictures of fretted instruments provided by contemporary musicians as well as from the historical images.

[8] Several twentieth-century musicologists, such as Mehdi Barkeshli (1912–1987), Hormoz Farhat (1928–2021), Sasan Sepanta (1934–2014) and Jean During (b. 1947), measured the intervals in different *dastgāh*s, as played by a few selected musicians, using electro-acoustic laboratory devices. Farhat (1990, 15) measured the intervals within the recorded musical pieces as played on two *tārs* and three *sitārs*, instead of analyzing vocal music as carried out by Barkeshli (1976, 84), reasoning that fretted instruments should provide a higher degree of stability of the pitches. However, the condition of the finger placement behind the fret (in terms of the actual distance and pressure) can affect the stability or repeatability of the pitch and may produce significant fluctuation in the intonation of a tone within a musical piece that involves fast movements of the fingers. On a fretted instrument, it would be simpler and more reliable or repeatable to measure intervals upon producing each tone one by one, as done in this study.

[9] Digital tuner applications on mobile devices are widely used by musicians and they could conveniently be used for taking measurements in this study. However, I needed to verify their accuracy and repeatability prior to using them in the study. To this end, a simple test was carried out using the mobile application Soundcorset Metronome and Tuner (v. 6.86, A4 = 440 Hz). This "app" shows deviations in cents from the 12-tone equal temperament (ET) tones. First, selected tones produced by a Casio PX-735 digital piano were evaluated and then the test was performed on a *tār* with a fretting system conforming to the 12-tone ET for the corresponding tones. In each test, the selected tone was produced twelve times and the deviations were recorded. On the piano, the sustain pedal was pressed upon each strike and released before the next strike. On the *tār*, the finger placement was kept for every four strikes and then the finger was lifted and placed again behind the same fret for the next set of strikes. In addition, the produced sound was damped with the hand before each strike.

[10] Table 2 shows the results of the two tests. On the piano, the produced tones showed a deviation range (i.e., maximum minus minimum) of ca. 1–4 cents for each tone with a deviation average of ca. 0–4 cents from the five selected reference tones and a standard deviation of ca. 1 cent for each tone. On the *tār*, the deviation range for each tone was wider (ca. 3–8 c) and the average values and the standard deviations of the deviations from the reference tones were slightly larger (ca. 2–5 c and ca. 1–3 c, respectively) than those encountered on the piano. Overall, it was concluded that the digital tuner app provides adequate accuracy and repeatability for the measurements, even on a hand-made traditional instrument with thread-type frets (rather than rigid metal frets as used in a guitar, for instance). It should be added that there are three double-string courses on a *tār*, typically tuned to C4–C4, G3–G3 and C4–C3).[6] It is essential that the double strings of the first course are tuned precisely in unison to avoid instability of the produced pitches. (±3 c of difference may not be an issue in my experience.) A precise tuning of all strings (even those not struck) is also desirable for the best results, although perhaps not necessary.

[11] In the late twentieth century, Majid Kiani (1992, 182) reported a pictorial measurement of the neutral intervals from photos of the *tārs* in the hands of two very renowned musicians

6. The strings of the first two courses are always tuned in unison. For the third course, they are usually tuned in octave, although other intervals (e.g. perfect fourth or neutral third) may be used in certain *dastgāhs* (Khaleghi 2012, 66-77).

taken about a century earlier. However, the accuracy of the pictorial method may be questioned, particularly due to the perspective effect (that could make the distances between the frets away from the center of the photo appear smaller than similar distances in the

Strike	Digital Piano						*Tār*					
	G3	A3	C4	G4	A4	C5	G3	A3	C4	G4	A4	C5
1	0	4	1	1	2	4	4	3	2	3	1	8
2	0	4	0	1	1	4	5	4	4	3	1	5
3	1	4	1	2	2	5	5	2	4	1	5	5
4	1	3	0	2	2	2	4	7	3	1	6	7
5	1	3	0	2	1	3	5	9	4	3	6	2
6	0	4	0	1	2	3	5	5	5	1	6	4
7	-1	4	0	1	2	2	3	4	4	2	6	4
8	-1	5	0	2	2	3	5	4	3	3	4	2
9	0	4	0	2	1	3	6	3	6	2	0	5
10	0	3	2	2	1	2	4	3	3	5	1	0
11	0	4	0	2	2	2	5	3	4	3	0	3
12	-2	4	1	1	2	1	5	3	4	2	2	6
Minimum	-2	3	0	1	1	1	3	2	2	1	0	0
Maximum	1	5	2	2	2	5	6	9	6	5	6	8
Range	3	2	2	1	1	4	3	7	4	4	6	8
Average	0	4	0	2	2	3	5	4	4	2	3	4
Standard Deviation	0.9	0.6	0.7	0.5	0.5	1.1	0.8	2.0	1.0	1.2	2.6	2.3

Table 2. Digital tuner test results showing deviations in cents from the reference chromatic tones.

centre) and the conceivable effect of the finger position on the tension of the wire (which is likely another factor affecting the pitch). Therefore, I decided to run some experiments to evaluate the accuracy and reliability of the pictorial method.

[12] I conducted the first test on my own *tār* (Figure 1). Each interval was calculated from the simple equation $(2L_0-L_1)/(2L_0)$, where L_0 is the length of the octave position and L_1 is the length of the fret position of interest from the nut, as marked in Figure 2. Figure 3 (top) shows the results in terms of deviations from the interval sizes determined by the tuner. The deviations were 10 c maximum, with an average of ca. 4 c below the tuner-determined values and a standard deviation of ca. 4 c.

[13] As a second test of reliability, measurements were taken on the pictures provided by two experienced *tār/sitār* players, namely BN and MM (with approx. 18 years of experience each), after the procedure for accurate octave fret adjustment and picture taking was carefully reviewed with them. The deviations of the measurements from the tuner-determined values for the selected five frets are presented in Figure 3 (bottom) along with the corresponding deviations for my *tār*. The deviations for the two musicians were generally smaller than those in the first test. Overall, it was concluded that the pictorial calculation method could produce results that are practically close to those obtained by a tuner, which was quite surprising and promising.

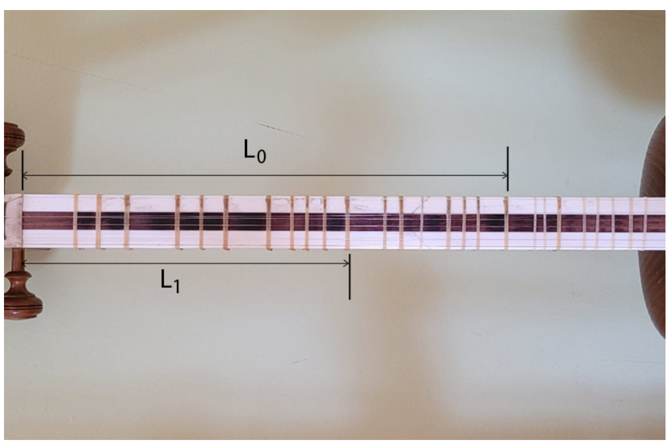

Figure 2. Pictorial interval measurement on the neck of a *tār*[7]

7. The original picture, rotated and cropped here to show the area of interest, was provided by one of the participants (AR) for pictorial interval calculations.

[14] Finally, the pictorial measurement was performed on two *tār* pictures published independently to guide learners how to adjust the frets. The first picture was a detailed drawing of a fretted *tār* neck (representing an 18-tone scale) at the beginning of a new edition of the main course book for the *tār* and *sitār* (Khaleghi 2012, 14) that is used in the *Hunaristān-i Mūsīqī-yi Millī* (the National Music School). The second picture was the photograph of a *tār* in Talai's hands (2017, 59) to show the *dastān-bandi* (fretting) of the instrument. Unlike in his 1993 publication, where Talai (1993, 18) provided rounded

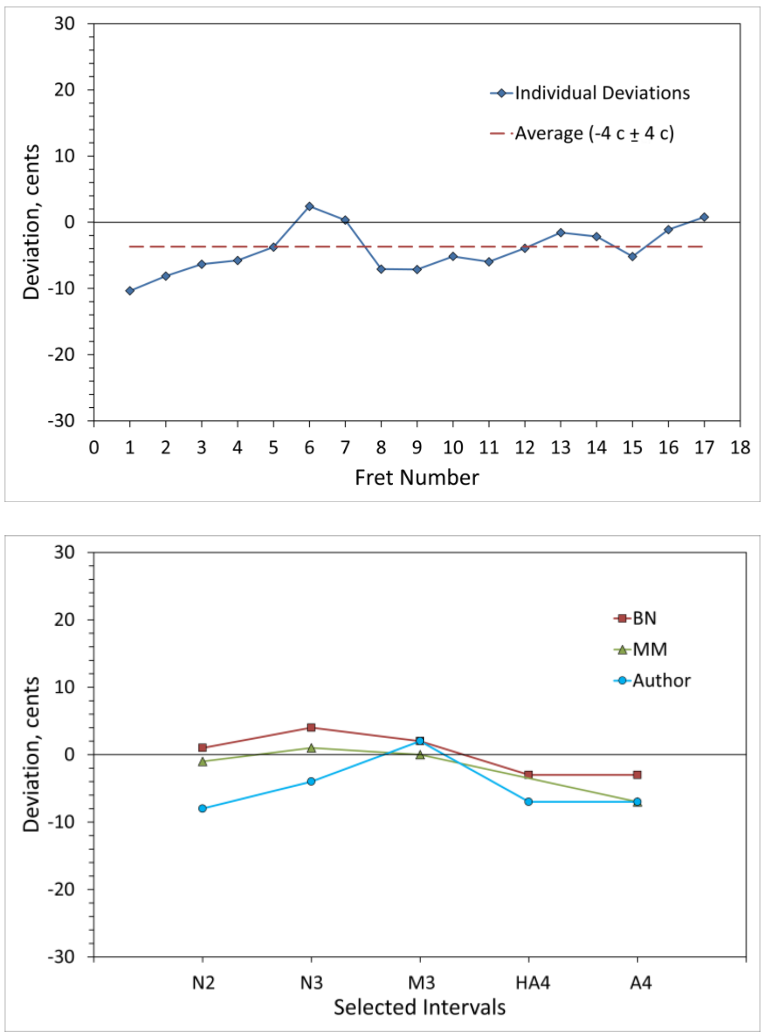

Figure 3. Deviations of the pictorially calculated intervals from those measured by a digital tuner: for all frets on my *tār* (top) and for selected frets by two participants (bottom).

Fret Position	Note	Khaleghi's *Hunaristān*, 18-tone		Talai, 16-tone		Kiani, 17-tone	
		Interval	**Step**	**Interval**	**Step**	**Interval**	**Step**
0	C	0.0	-	0.0	-	0.0	-
1	D♭	93.3	93.3	N.A.		90.2	90.2
2	D𝄳	148.9	55.6	148.9	148.9	143.5	53.3
3	D	204.6	55.7	202.5	53.7	203.9	60.4
4	E♭	294.7	90.2	292.8	90.3	294.1	90.2
5	E𝄳	351.8	57.0	348.6	55.8	354.5	60.4
6	E	410.7	58.9	398.5	49.8	407.8	53.3
7	F	498.5	87.8	499.0	100.5	498.0	90.2
8	F𝄲	542.1	43.6	N.A.		558.5	60.4
9	F#	587.9	45.8	575.5	76.5	N.A.	
10	G𝄳	648.0	60.1	644.4	68.9	641.5	83.1
11	G	702.3	54.3	704.7	60.3	702.0	60.4
12	A♭	788.9	86.6	793.3	88.6	792.2	90.2
13	A𝄳	849.2	60.2	844.4	51.1	845.5	53.3
14	A	904.0	54.8	905.3	60.8	905.9	60.4
15	B♭	997.6	93.6	980.5	75.2	996.1	90.2
16	B𝄳	1053.2	55.7	1050.2	69.7	1056.5	60.4
17	B	1106.5	53.2	1097.8	47.6	1109.8	53.3
18	C'	1200.0	93.5	1200.0	102.2	1200.0	90.2

Table 3. Pictorial interval measurements in cents for two published *tār* pictures along with Kiani's proposed interval sizes included for comparison. N.B. Some step sizes may not add up precisely to the interval values due to rounding.

interval sizes in cents for a 15-tone scale, he did not discuss the interval sizes or the fretting system in the 2017 publication explicitly and instead relied on a picture to show the fret positions. The results of the interval measurements on these two pictures are presented in Table 3, along with the 17-tone scale intervals proposed by Kiani[8] (1992, 192), for comparison. Not only were the results of the two pictorial measurements comparable for most fret positions, they were also close to those explicitly presented by Kiani for most tones.[9] All these evaluations prove the validity of the simple pictorial method presented here.

TESTS AND RESULTS

[15] A multiple-choice online survey was prepared in two parts and ten questions using Google Forms (Table 4). The aim of Part 1 was to collect information on the background of the participating musicians and their opinions on the variability and subjectivity of the practiced fretting systems as well as the need for one or more standardized fretting systems. Part 2 gathered quantitative information (using a digital tuner) on five selected intervals in the parent scale. The *tār/sitār* players were also asked to provide a full image of their instrument to be used for interval calculations. It was requested that the picture to be taken at a distance of ca. 1.5 m from the instrument and the camera face to be parallel to the finger board surface. A short introduction and instruction sheet was also prepared in Persian (including the translation of the questions that were in English) and was made available to the participants from Iran.

[16] The survey was sent to over fifty musicians, but only twenty-four musicians (including five *santūr* players) responded and participated in Part 1 and only nineteen musicians (including four *santūr* players) participated in Part 2. Table 4 lists the questions as well as the summary of the responses. The individual responses are provided in Appendix 1. The

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8. Kiani presented ratios for each tone as well as the interval sizes in savarts and cents. Some of the conversions in the original source were in error, but are corrected in Table 4.
9. An exception was for Talai's Fret Position 15 (B flat) with a calculated interval size of 980 c from the open string. This calculated size was consistent with the size Talai (1993, 18) reported earlier for a 15-tone scale. He had assigned an interval of 80 c above A natural (with a rounded size of 900 c). Most twentieth-century musicologists, such as Farhat (1990, 17), had assigned an interval of 90 c (i.e., approximately a limma) above A natural (with a size of ca. 905 c, producing a m7 of ca. 995 c).

Part	No.	Questions	Responses
1	1	Which string instrument is your main instrument?	33% *tār*, 46% *sitār* and 21% *santūr*
	2	How long have you been playing it?	2–35 y; 21 years on average
	3	Do you rely on a tuner to tune the strings/chords of your instrument or adjust the frets?	8% never, 42% sometimes, 38% often and 13% always
	4	What do you think about the following statement? Iranian musicians adjust the frets (on a *tār/sitār*) or tune the strings (on a *santūr*) somewhat differently (esp. with respect to the neutral tones, such as *mī kurun* and *fa surī*) and subjectively (e.g., based on what they have been taught or based on how the notes sound to their ears), instead of following a standardized guideline or fretting system.	67% agree, 8% disagree and 25% neither agree nor disagree
	5	What do you think about the following statement? It would be very useful to have at least one standardized guideline or fretting system to use for organizing the frets (e.g., on a *tār/sitār*) or tuning the strings (e.g. on a *santūr*), using a digital tuner.	79% agree, 4% disagree and 17% neither agree nor disagree
2	6	For the next questions you need a digital tuner. There are many free apps that can be installed on your mobile phone. Remember that there is no correct or wrong answer. First, tune your first string on the *tār* (*zīr* or *do*) exactly on C4 using the digital tuner. Play *re kurun* (D half flat). What is the approximate deviation from the shown tone (C#/D ♭ or D natural) on the tuner?	140–160 c; **148** (± 6) c
	7	Play *mi kurun* (E half flat). What is the approximate deviation from the shown tone (D#/E ♭ or E natural)?	340–365 c; **349** (± 6) c
	8	Ensure your open string (*do*) is still exactly on C4. Play *mi bécarre* (E natural). How much of deviation does it show from E4 on the tuner approximately?	375–405 c; **398** (± 7) c
	9	Play *fa surī* (F half sharp). How much of deviation does it show from F4 or F#4/G ♭4 on the tuner approximately?	520–575 c; **549** (± 12) c
	10	Play *fa dièse* (F sharp). How much of deviation does it show from F#4 or G ♭4 on the tuner approximately?	590–620 c; **603** (± 8) c

Table 4. Survey questions and summary of the responses. Number of participants: 24 in Part 1 and 19 in Part 2; For the questions in Part 2, the answers (see Appendix 1) were converted to the intervals from the open string tone in the following format: minimum–maximum; **average** (± standard deviation).

participants had a wide range of experience (from 2 years to 35 years, approximately). The majority of the musicians would use a digital tuner for tuning the strings or adjusting the frets at least occasionally and only 8% would never use a tuner (solely relying on their ears). Most participants agreed that the practice of tuning and fretting in Iran is rather subjective and non-standardized (67%) and agreed that having one or more standardized systems would be beneficial (79%).

[17] I followed up with some participants by email (or other media) in order to confirm or discuss their answers. I was particularly interested in talking with those who expressed disagreement with the statements in Questions 4 and 5 and finding their reasons. Some participants were easier to communicate with than others. Upon checking on the question of subjectivity of the fretting systems and the lack of a standardized system (Question 4) with three of the participants that disagreed initially (i.e., SV, AA1 and AR with approx. 2, 8 and 28 years of experience respectively), I found that the point of the question was not clear to them. After explaining the question in more detail, they changed their responses. Upon discussing the matter with some other participants who either disagreed (e.g., HE) or "neither agreed nor disagreed" (e.g., BN), one of the following points were brought up:

> 1) they believed that there were one or a few fretting systems but they were not aware or could not agree that such systems were not necessarily followed by most practicing musicians. I explained to them that there are actually several tuning systems and the results of the survey show that majority of the musicians adjust the frets (particularly the neutral tones) subjectively.
>
> 2) they were not aware or could not agree that none of such systems were considered a standard system in Iran. I explained that there is actually no standard fretting system for Persian art music.
>
> 3) they could not agree that the fretting or tuning of the instruments is done in a "subjective" manner. From the conversations, I suspected that the concept of subjectivity was not very clear to a small group of participants.

[18] In Part 2, the participants were asked to tune the lower string(s) of their *tār/sitār* (referred to as *zīr*) exactly on C4 using a digital tuner and report the deviations (in cents) of five selected tones from the reference tones on the tuner. Part 2 of Table 4 shows the

summary of the results after the reported deviations were converted to the interval sizes from the open string. The selected intervals comprised neutral second (N2), neutral third (N3), major third (M3), half-augmented fourth (HA4) and augmented fourth (A4). The results showed a range of ca. 20–30 c for all the intervals, except for the HA4 that showed a range of ca. 55 c. F half-sharp is very variable among musicians and some only use one fret for both F half-sharp and F sharp (Farhat 1990, 17).

[19] Nevertheless, as shown in Figure 4, the average of all reported measurements practically conformed to the corresponding values of a 24-tone ET parent scale (i.e., 150, 350, 400, 550 and 600 c.), proposed by Ali Naqi Vaziri (1886–1981) in the early twentieth century (Vaziri 1934, Part 2, 4–10). The mean reported values ranged from 2 c below to 3 c above the corresponding ET values with an average of 1 cent below. Such a small difference is almost below the "difference limen" or "just-noticeable difference" for most trained and sensitive musicians, which is 2–3 c (Parncutt and Cohen 1995, 836). Figure 4 also shows that the average *santūr* results were generally close to the average *tār/sitār* results (with the largest deviation being on the contentious interval of HA4). Therefore, including or excluding the few *santūr* results from the rest of the results would not make any significant difference in the findings, as apparent in Figure 4 that shows similar deviations with and without the *santūr* results.

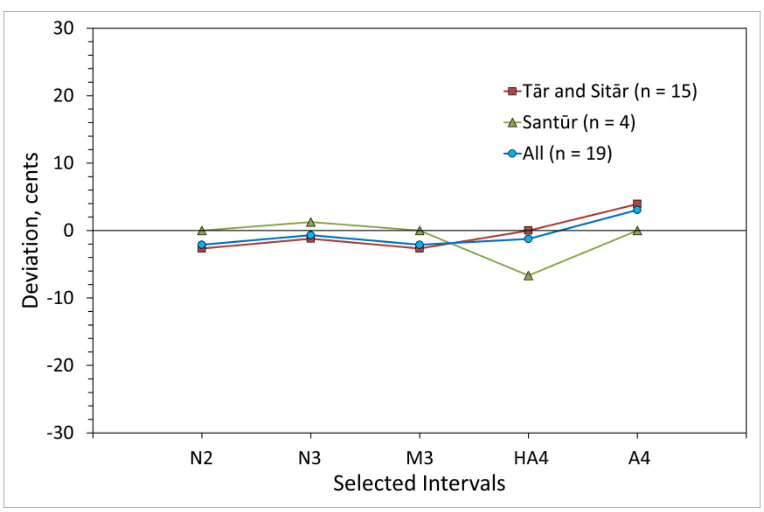

Figure 4. Deviations of the average reported interval sizes from the corresponding 24-tone ET values.

[20] In general, some errors could be associated with the reported results if the open string on the *tār/sitār* was not tuned exactly on C4 at the time of each measurement or if the multiple-choice answer was marked incorrectly (which is a possible source of error in any survey). Using a *santūr*, an instability of the measured pitch may occur if all the four strings for each course are not tuned exactly in unison. This source of error is less pronounced for a *tār* (with double-string courses) and non-existent for a *sitār* (with single strings).

[21] I received a total of ten pictures of *tārs/sitārs* from the participants (excluding mine), but only nine of them were accompanied with tuner measurements to be used for comparison. The pictorial measurements were carried out on all pictures received. The individual measurements are provided in Appendix 2 and the summary of the results is presented in Figure 5. The average of all the results based on the pictorial calculations for the selected five pitches (the shaded columns in Table 5) were very close to the average of the results based on the tuner measurements that were reported by the participants (Table 4). This is also evident

Note	**D♭**	**D𝄳**	**D**	**E♭**	**E𝄳**	**E**	**F**	**F𝄲**	**F#**	**G𝄳**	**G**	**A♭**	**A𝄳**	**A**	**B♭**	**B𝄳**	**B**
Fret No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
n*	6	10	10	10	10	10	10	7	9	9	10	10	10	10	10	10	10
Min.	88	129	192	280	335	390	486	537	576	623	683	762	834	891	985	1043	1091
Max.	104	149	211	303	364	417	521	561	613	665	722	815	872	916	1013	1062	1112
Range	16	20	19	23	29	27	35	24	37	42	39	53	38	25	28	19	21
Ave.	**97**	**140**	**200**	**293**	**344**	**399**	**498**	**550**	**595**	**640**	**700**	**790**	**848**	**905**	**996**	**1052**	**1103**
St. Dev.	5	7	6	6	9	8	11	8	10	13	11	14	11	8	8	6	6

Table 5. Intervals in cents as determined by the pictorial method.
*The number of participants (n) could be less than 10 for some tones, as some instruments lack the corresponding frets. The shaded columns mark the intervals selected in the survey (to be compared with the results in Part 2 of Table 4).

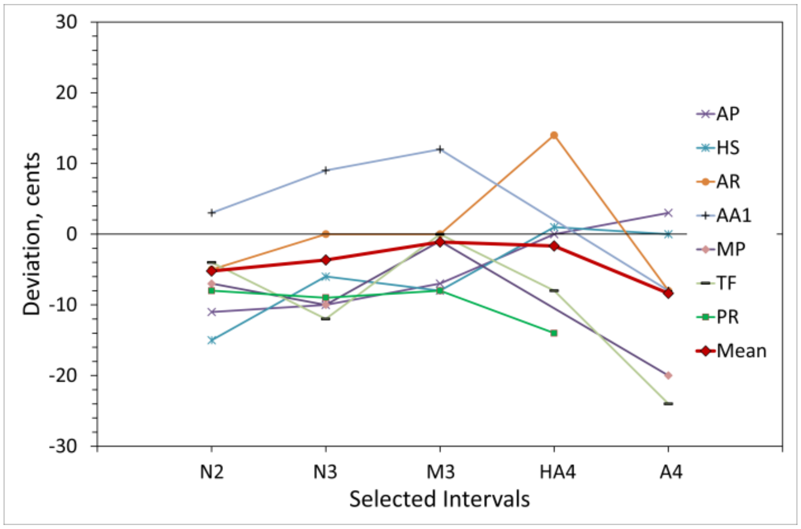

Figure 5. Deviations of the pictorially calculated selected intervals from those measured by a digital tuner for all participants except those shown in Figure 4.

from the "mean" curve[10] in Figure 5 that lies close to the zero line. However, the deviations between the two methods for the individual participants could be significantly large in some cases, as apparent in Figure 5. Possible reasons for such large deviations could include inaccurate tuning of the open string and/or fretting of the octave tone, change in the position of some of the frets between the tuner test and the time of picture taking (as some musicians move the frets between solo and group performances or for playing different *dastgāh*s) and errors in filling out the survey.

[22] As stated above, the average values from the two methods were close for the selected five tones and they were also close to the ET intervals, as demonstrated in Figure 6. The maximum deviation of the average values from the ET intervals was 10 c, which is quite small. These results support the proposal of a theoretical parent scale either with a small deviation of the tones from the corresponding ET values (e.g., 10 c maximum) or with a temperament conforming to the ET values. An important conclusion that can be drawn from the empirical results is that, in practice and on average, the neutral second and third are approximately 140–150 c above the open string and major second, respectively, and the half-augmented fourth is approximately 50 c above the perfect fourth.

10. The mean curve represents the average of pictorial measurements for all participants, excluding mine.

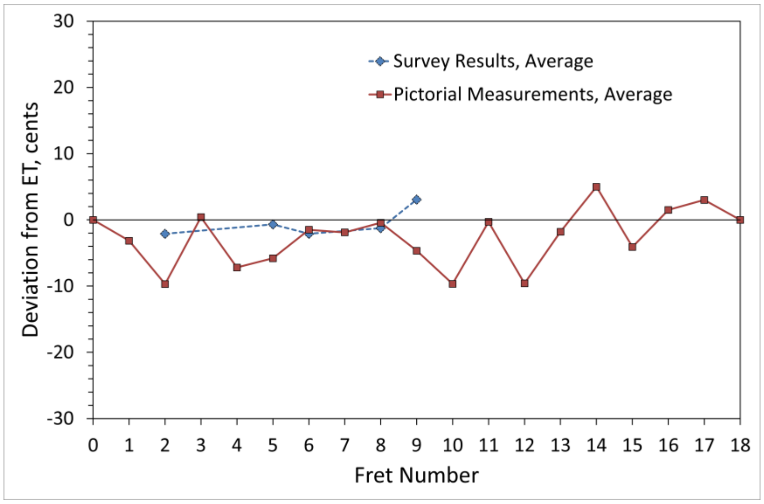

Figure 6. Deviations of the average of reported and pictorially calculated intervals from the corresponding intervals in a 24-tone ET scale.

Proposed Parent Scales

[23] In the previous study (Poorhaydari 2022, 43–47), two theoretical approaches were adopted to determine a neutral second interval that is close to the practice (i.e., approximately 140–150 c): an indirect approach and a direct approach. Those approaches are briefly reviewed and amended here. Through the indirect approach, several sub-tone intervals were examined to lower/raise a main tone and obtain the neutral tone. The most promising sub-tone interval examined was the one calculated halfway between a limma and a comma (i.e. (L+C)/2), with the size of ca. 56.8 c,[11] and resulting in a neutral second of ca. 147.1 c. I would like to call this sub-tone interval a *nīmā* (N).[12] Based on this proposal, a whole tone would have an internal structure of LNN, instead of LLC or LCL. Through the direct approach, an arithmetic mean calculation (an ancient tradition that was also used during the early Islamic era) was adopted to determine a *mujannab dastān* midway between *muṭlaq* (the open string) and Ibn Sina's *wusṭā fārs* (also known as *wusṭā qadīm*, i.e., the

11. The geometric mean between a limma and a comma yields an interval size of $\sqrt{\frac{2187}{2048}}$ (equivalent to 56.84 c), whereas the arithmetic mean between the two Pythagorean intervals yields a ratio of 272,097,792:263,357,891 (equivalent to 56.52 c). The former is more accurate and consistent with the logarithmic calculations in cents (i.e., the average of 90.22 c and 23.46 c).

12. The proposed name comes from the Persian word *nīm* meaning half.

Pythagorean minor third) of 32:27 (294.1 c),[13] which yielded a ratio of 64:59 (140.8 c). This *mujannab*, which can be referred to as *mujannab-i fārs* (Persian neutral second), is exactly a whole tone (9:8, 203.9 c) below Ibn Sina's *wusṭā zalzal* of 72:59 (344.7 c).

[24] To calculate perfectly equal neutral intervals, a geometrical mean calculation through root extraction would be needed, which yields an interval of $\sqrt{32:27}$, equivalent to 147.1 c, similar to the result of the indirect approach. In fact, it can be shown through simple mathematical relationships that the above-mentioned direct approach yields the proposed sub-tone interval of N.[14] Therefore, the proposed theoretical *mujannab* would approximately be either 141 c or 147 c, depending on the method of averaging (arithmetic or geometric, respectively).

[25] Figure 7 shows the proposed *nīmā*-based *dastān* positions and lengths in a tetrachord, based on three criteria: arithmetic mean calculation, geometric mean calculation and tempering. The arithmetic mean calculations result in two values for the *nīmā* in a whole tone, a smaller *nīmā* (N_1 of 243:236 or 50.60 c) and a larger *nīmā* (N_2 of 531:512 or 63.08 c), whereas the whole tone established through the geometric mean calculation comprises two units of the mean *nīmā* (N of 56.84 c each). In the tempered scale, the mean *nīmā* is reduced to 55 c upon the reduction of the whole tone from 204 c to 200 c. The difference between the three criteria is mainly of theoretical significance and can be considered practically small. The maximum difference between the calculated and tempered positions is 8 cents (i.e. for E). This is the difference between the ET and the Pythagorean major thirds.

[26] Another candidate sub-tone interval mentioned in the previous study, but not used in the proposed scale, was a diesis or a quartertone. Here, this non-Pythagorean interval is examined to present an alternative model for an 18-tone scale. In Greek music (Mathiesen

13. It should be added that after the initial proposal (Poorhaydari 2022, 46) I discovered that this *mujannab* was mentioned by the Systematists (Shirazi 2008, 116), although not elaborated on or favoured, as one of the three *mujannab dastān*s that the practitioners would establish between two other *dastān*s in the early Islamic time.
14. The *wusṭā qadīm* has a length of (2L+C) + L (i.e., 9:8 x 256:243 = 32:27). The midway position between *muṭlaq* and *wusṭā qadīm* will divide this length in two, resulting in a *mujannab* position of L + (L+C)/2, which is (L+C)/2 short of a *ṭanīnī* of 2L+C.

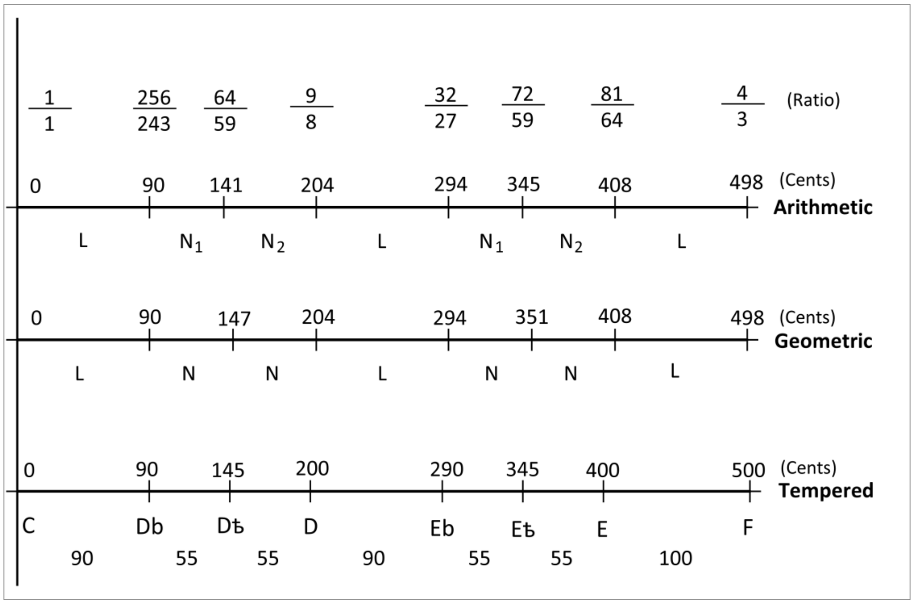

Figure 7. Proposed *dastān* positions in a tetrachord using *nīmā* intervals.

2002, 116), a whole tone of 9:8 could be divided into two semitones of 18:17 and 17:16 or into four dieses or quartertones of 36:35, 35:34, 34:33, and 33:32. Therefore, the candidate intervals for a minor second (combining the first two dieses) and a neutral second (combining the first three dieses) would be 18:17 (99.0 c) and 12:11 (150.6 c), respectively.[15] The other minor and neutral intervals can simply be calculated by arithmetic addition of these two intervals to the lower main tones, as shown in Figure 8 for the intervals in a tetrachord along with their tempered values.

[27] In order to calculate the positions of all frets within an octave (or even along the entire neck of a *tār*/*sitār* that is typically up to a minor thirteenth, e.g. C4 to A♭5), one could extend the above-mentioned *dastān* positions in the tetrachord to the octave, since an octave

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15. A *dastān* of 18:17 was referred to as *mujannab al-sabbāba bi tanṣīf al-ṭanīnī al-awwal* (adjacent to the index finger by halving the first whole tone) by the Systematists (Shirazi 2008, 120). A neutral second of 12:11 can also be obtained by calculating the arithmetic mean between a minor second of 18:17 and a major second of 9:8. It is worth noting that in the eleventh century Ibn Sina assigned the ratios of 16:15 (111.7 c) and 13:12 (138.6 c) for the minor second and the neutral second on the ancient Iranian instrument *rubāb* (Barkeshli 1976, 74), which are somewhat different from the proposed ratios here.

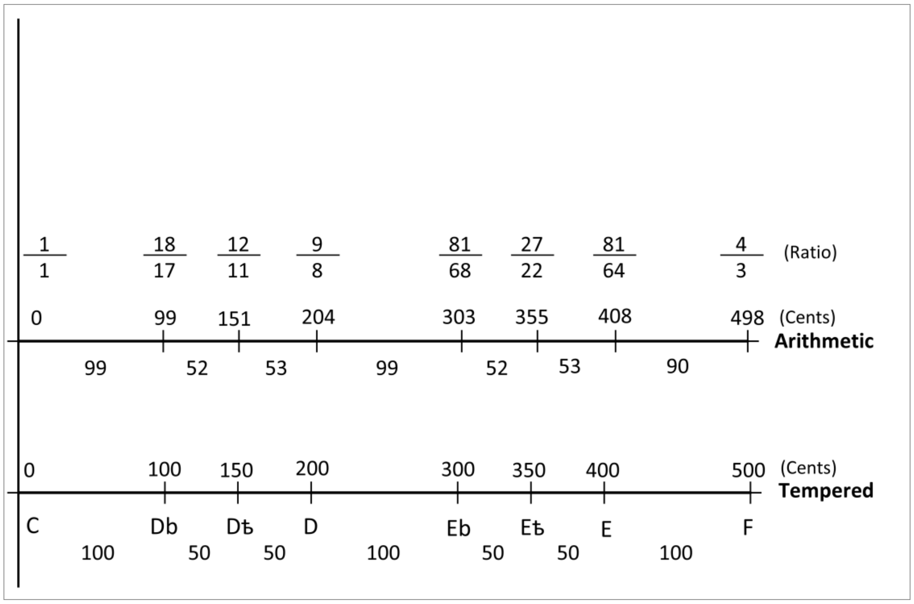


Figure 8. Proposed *dastān* positions in a tetrachord using diesis intervals.

comprises two tetrachords and a whole tone (placed between the two tetrachords in the proposed scales, i.e., using a disjunct tetrachord arrangement). Table 6 presents the proposed 18-tone fretting calculations, based on the two theoretical models. Two scales are presented for the first model, one using the proposed *mujannab* of 64:59 (resulting in two unequal *nīmā*s of ca. 50.6 c and 63.1 c) and the other using the mean *nīmā* of ca. 56.8 c. The second model is based on dieses or quartertones. The scales in both models have a diatonic backbone with Pythagorean tuning (Herlinger 2002, 171), in which the seven tones are separated by a Pythagorean whole tone (9:8) or a Pythagorean semitone (256:243). The other tones (i.e., flats, half-flats and the half-sharp) have different values in the three scales, based on the type of sub-tone interval used. In the first model, all the flat tones are a limma (90.2 c) above the lower tones, whereas in the second model they are 18:17 (99.0 c, practically consistent with a semitone) above the lower tones. The half-flat tones are 64:59 (140.8 c) and ca. 147.1 c above the lower Pythagorean tones in the first and the second scales of the first model, respectively, and 12:11 (150.6 c) above the lower Pythagorean tones in the second model. The half-sharp tone above the perfect fourth (F‡) is set to be a "*mujannab-i fārs*" (140.8 c or 147.1 c) above E in the first model but, following the diesis-based structure of the whole tone, it is 36:35 (48.8 c) above F in the second model.

Fret Position	Note	Based on a Limma-Comma Function (Midway Calculation)							Based on Dieses (Quartertones)				
		Using the Proposed *Mujannab* of 64:59			Using the Mean *Nīmā* of 56.8 c		As Tempered		As Calculated			As Tempered	
		Ratio	Interval	Step	Interval	Step	Interval	Step	Ratio	Interval	Step	Interval	Step
0	C	1:1	0.0	-	0.0	-	0	-	1:1	0.0	-	0	-
1	D♭	256:243	90.2	90.2	90.2	90.2	90	90	18:17	99.0	99.0	100	100
2	D𝄳	64:59	140.8	50.6	147.1	56.8	145	55	12:11	150.6	51.7	150	50
3	D	9:8	203.9	63.1	203.9	56.8	200	55	9:8	203.9	53.3	200	50
4	E♭	32:27	294.1	90.2	294.1	90.2	290	90	81:68	302.9	99.0	300	100
5	E𝄳	72:59	344.7	50.6	351.0	56.8	345	55	27:22	354.5	51.7	350	50
6	E	81:64	407.8	63.1	407.8	56.8	400	55	81:64	407.8	53.3	400	50
7	F	4:3	498.0	90.2	498.0	90.2	500	100	4:3	498.0	90.2	500	100
8	F𝄲	81:59	548.6	50.6	554.9	56.8	545	45	48:35	546.8	48.8	550	50
9	F#/G♭	1024:729	588.3	39.6	588.3	33.4	590	45	72:51	597.0	50.2	600	50
10	G𝄳	256:177	638.9	50.6	645.1	56.8	645	55	16:11	648.7	51.7	650	50
11	G	3:2	702.0	63.1	702.0	56.8	700	55	3:2	702.0	53.3	700	50
12	A♭	128:81	792.2	90.2	792.2	90.2	790	90	27:17	800.9	99.0	800	100
13	A𝄳	96:59	842.8	50.6	849.0	56.8	845	55	18:11	852.6	51.7	850	50
14	A	27:16	905.9	63.1	905.9	56.8	900	55	27:16	905.9	53.3	900	50
15	B♭	16:9	996.1	90.2	996.1	90.2	990	90	243:136	1004.8	99.0	1000	100
16	B𝄳	108:59	1046.7	50.6	1052.9	56.8	1045	55	81:44	1056.5	51.7	1050	50
17	B	243:128	1109.8	63.1	1109.8	56.8	1100	55	243:128	1109.8	53.3	1100	50
18	C'	2:1	1200.0	90.2	1200.0	90.2	1200	100	2:1	1200.0	90.2	1200	100

Table 6. Proposed 18-tone parent scales based on two models. All interval and step sizes are in cents. Some step sizes may not add up precisely to the interval values due to rounding.

[28] The tempered values of the intervals are also provided for all fret positions in Table 6. The maximum difference between the calculated values of the intervals and the tempered values is ca. 10 c (Fret Position 17), which is the difference between the Pythagorean and ET major seventh intervals. An important characteristic of both models is that all successive similar tones (e.g., D ♭–E ♭ and D 𝄳–E 𝄳) are separated exactly by a whole tone (9:8 or 203.9 c that was rounded down to 200 c in the tempered scales), as was the case for the backbone Pythagorean diatonic tones (e.g., D–E). This characteristic makes key transposition possible with the least (if any) distortion of the scale intervallic structure. It also produces a larger number of perfect fourth and fifth intervals between the scale degrees, which was a criterion for a higher degree of consonance of a mode according to the Systematists.[16]

[29] It should be added that a slightly different version of the Pythagorean limma-comma-based parent scale was initially proposed (Poorhaydari 2022, 50). The difference was in the neutral tones, which were calculated midway between the immediately lower and upper tones, using arithmetic mean calculations. The calculated neutral intervals were very close to those in the second scale of the first model in Table 6, except for the eighth position (F𝄲) that was calculated to be ca. 543.2 c (which is closer to that of the first scale of the first model). The scale based on the midway calculation resulted in practically identical intervals as calculated pictorially from the drawing in Khaleghi's *hunaristān* course book (Table 4). The maximum deviation of the intervals in the two systems was 4 c, which is negligible. This observation suggests that the accurate drawing was likely based on the Pythagorean-based intervals with midway calculations for the neutral tones. Of particular note is the size of the minor intervals in the drawing that is close to a limma (90 c), rather than a semitone of 18:17 (99 c). This, being consistent with the corresponding intervals proposed by the Systematists as well as the twentieth-century musicologists (Poorhaydari 2022, 50), shows the historical

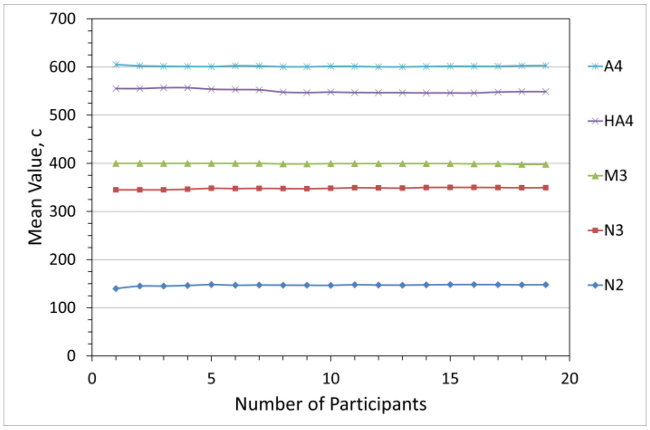
16. According to Urmawi (2001, 24), there are two types of *tanāfur* (dissonance) for a *dawr* (cycle/mode): *ẓāḥir* (visible) and *khafīh* (invisible). The *dawr* has visible dissonance when one of the four specified arrangements of the small intervals (*baqiyya*, *mujannab* and *ṭanīnī*) occurs. (The specified arrangements are not discussed here.) The invisible dissonance occurs when the total number of *nisab al-sharīf* (prominent intervals, i.e. the perfect fourth, fifth and eighth) in the cycle is less than the number of tones in that cycle. Therefore, the degree of consonance increases with the increasing number of achievable perfect fourths and fifths in a mode.

importance of a Pythagorean semitone (and generally the Pythagorean system) in Persian art music theory.

FURTHER DISCUSSION AND CONCLUSION

[30] After the analysis was completed, most participants were approached in order to receive feedback. Three participants (AP, AR, and PR) accepted the invitation and the results of the survey as well as the two proposed models were presented to them through video calls. They all considered the results extremely important and beneficial for ensemble playing. In particular, PR expressed grief on deciding the fret positions (especially with respect to the neutral tones) on each session of ensemble practice (whether to prepare for a concert or to produce a recording). Additionally, they were very satisfied with the proposed models. AR pointed out that the proposed models have historical, theoretical as well as practical justifications. To the best of my knowledge, the combination of all these aspects has not been attempted in the past to the extent achieved in this two-part investigation (beginning with the first article and continued in the present article).

[31] A question may arise regarding the effect of the number of participants on the results (particularly the intervals) and the conclusions. It was hoped that over 50 musicians (with a wide range of experience) would participate in the survey. However, only 24 musicians participated in Part 1 and only 19 participated in Part 2. In addition, only 10 musicians, from a total of 19 *tār/sitār* players, provided pictures of their instruments for pictorial measurement. Although the measured intervals varied significantly among the participants, the mean values showed practically small fluctuations with respect to the number of participants, as shown in Figure 9 for the tuner-based responses. In the left-side diagram, the number of participants are in the order the responses were received, starting with BN and ending with KM for the tuner measurements. A different order of analyzing the responses (e.g., based on the increasing years of experience) would change the fluctuations at the beginning of the graphs, as shown in the right-side diagram of Figure 9 for one of the selected intervals, but not toward the end, where the graphs tend to change magnitude slightly and steadily. Similar fluctuations were observed for the pictorial measurements, but the diagrams are not presented here for briefness.



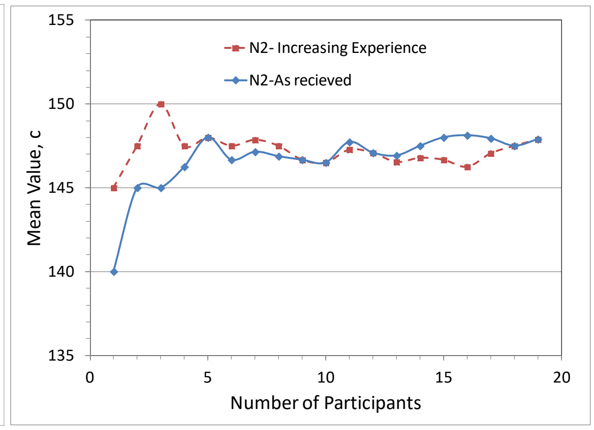


Figure 9. Variations of the average values of the tuner measurements with the increasing number of participants for the five selected intervals in the order the responses were received (left) and for the first selected interval in two different orders (right).

[32] It should be added briefly here that a recent publication on computational analysis of Iranian music intervals by Shafiei (2021, 12) also reported a relatively wide range of measured intervals. The intervals were extracted from the vocal pieces in *dastgāh-i shūr* performed by master Mahmud Karimi (1927–1984). A simple statistical analysis (not presented by Shafiei) shows that the 46 reported measurements for a neutral second interval in that analysis (formed between different degrees) ranged from 120 c to 165 c and had a mean value of 143 c and a standard deviation of 13 c. The mean value of 143 c is consistent with the conclusion in this study on the average size of the neutral second in practice (i.e., 140–150 c), derived from the experimental measurements as well as the twentieth-century literature review.

[33] The participants had a wide range of experience. However, no reliable correlations could be made between the survey responses and the years of experience of the participants. (Whether or not some correlations could be made if a larger number of musicians had participated in the survey remains unanswered.) An exception might have been regarding the use of a tuner. The two participants who replied "never" had over 30 years of experience (MG and MP; see Appendix 1). They are from a generation of music learners in Iran who did not have a convenient access to digital tuners and they, similar to their instructors, would rely on their ears for "proper" tuning or fret adjustments. However, several studies have shown that even advanced musicians may not be able to recognize or produce a given

interval accurately and there could be significant fluctuation in intonation from the intended tones/intervals. Rakowski and Miskiewicz (1985) asked music students to tune a machine (analogous to a musical instrument) at certain intervals from a reference tone. The melodic intervals were initially provided to them audibly and they were asked to tune the machine accordingly. They found large variations in intonation in the responses (i.e., deviations as large as ± 20 c between the tuned intervals and the expected ET intervals that were initially provided to them). Vurma and Ross (2006) examined the production of selected intervals by professional vocalists and the perception of those intervals by expert listeners. They found the produced melodic intervals may have on average ± 20–25 c deviation from the expected ET intervals and yet be considered properly tuned by advanced listeners. These findings indicate that relying on ears (even for advanced or professional musicians) can result in significantly inaccurate intervals.

[34] The emphasis of this article is on the need for the standardization of the scale, particularly with respect to the neutral intervals. Undoubtedly, clearly defined intervals in a parent scale are fundamental to a music theory, and ratios (although questioned by some musicologists)[17] or interval sizes in cents provide an objective basis for music theorists as well as the practitioners. In addition, having intervals of variable sizes in different modes negates the idea of having a systematized parent scale, which has been the objective of the Systematists as well as twentieth-century scholars. The survey conducted as part of this study also showed that establishing a systematized parent scale is favoured by the majority of the contemporary practitioners. It is likely a matter of perspective that some musicologists only describe the characteristics of Persian art music and consider the variability as a flexibility and part of the identity of this traditional practice, while others—especially the younger practicing musicians—seek for simplicity, consistency, and a standardized theoretical foundation.

17. The ratio theory (e.g., Pythagorean or just) has been questioned by some "modernist" musicologists (Parncutt and Hair 2018, 492), especially from the psycho-cultural point of view that is more in support of subjectivity than objectivity (mathematics or physics). They suggest that "all kinds of interval, scales, and harmonies are musically possible." This last statement is in line with the suggestion in this article that the exact size of the neutral interval (140, 145, 150, etc.) is not important, yet for the sake of ensemble music performance and consistency in Persian art music at least one size should be agreed upon and standardized.

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APPENDIX 1: INDIVIDUAL SURVEY RESULTS

Participant	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
HE	*Santūr*	< 3 y	Always	Disagree	Agree	Not completed by the participant				
SV	*Tār*	< 3 y	Always	Agree	Agree	Not completed by the participant				
AA1	*Sitār*	6-10 y	Sometimes	Neither	Agree	45 c above C#/Db	45 c below E	5 c above E	45 c below F#/Gb	10 c above F#/Gb
TF	*Sitār*	6-10 y	Often	Agree	Agree	50 c below D	50 c below E	10 c below E	45 c above F	Almost no deviation
SA	*Tār*	12 y	Sometimes	Agree	Agree	45 c below D	45 c below E	Almost no deviation	45 c above F	Almost no deviation
BN	*Tār*	16-20 y	Often	Neither	Agree	40 c above C#/Db	45 c above D#/Eb	Almost no deviation	45 c below F#/Gb	5 c above F#/Gb
PA	*Sitār*	16-20 y	Sometimes	Agree	Disagree	50 c below D	45 c above D#/Eb	Almost no deviation	45 c below F#/Gb	Almost no deviation
EA	*Sitār*	16-20 y	Often	Agree	Agree	45 c above C#/Db	45 c above D#/Eb	Almost no deviation	40 c below F#/Gb	Almost no deviation
MM	*Sitār*	16-20 y	Most of the time	Agree	Agree	50 c below D	50 c below E	Almost no deviation	No *fa suri* fret	Almost no deviation
SM1	*Sitār*	16-20 y	Sometimes	Neither	Neither	45 c above C#/Db	45 c above D#/Eb	10 c below E	20 c above F	10 c below F#/Gb
AS	*Santūr*	16-20 y	Often	Agree	Neither	40 c above C#/Db	45 c above D#/Eb	Almost no deviation	45 c above F	10 c below F#/Gb
FK	*Santūr*	16-20 y	Often	Neither	Agree	45 c above C#/Db	45 c above D#/Eb	Almost no deviation	45 c above F	Almost no deviation

MI	*Sitār*	16-20 y	Always	Agree	Agree	45 c below D	35 c below E	Almost no deviation	40 c above F	10 c above F#/Gb
PR	*Sitār*	16-20 y	Sometimes	Agree	Agree	45 c above C#/Db	45 c above D#/Eb	Almost no deviation	25 c below f#/Gb	No *fa dièse* fret
AP	*Tār*	21-25 y	Sometimes	Agree	Agree	40 c above C#/Db	45 c above D#/Eb	Almost no deviation	50 c above F	10 c above F#/Gb
HS	*Tār*	26-30 y	Often	Agree	Agree	50 c below D	50 c below E	Almost no deviation	below F#/Gb	Almost no deviation
AR	*Tār*	26-30 y	Often	Agree	Agree	45 c above C#/Db	45 c above D#/Eb	Almost no deviation	40 c above F	Almost no deviation
SR	*Tār*	30 y	Sometimes	Agree	Agree	Not completed by the participant				
MG	*Sitār*	32 y	Never	Disagree	Agree	Not completed by the participant				
FT	*Sitār*	31-40 y	Sometimes	Agree	Agree	Not completed by the participant				
SM2	*Tār*	31-40 y	Sometimes	Neither	Agree	40 c above C#/Db	40 c above D#/Eb	25 c below E	45 c below F#/Gb	20 c above F#/Gb
MR	*Santūr*	31-40 y	Often	Neither	Neither	40 c below D	40 c below E	Almost no deviation	40 c above F	Almost no deviation
MP	*Tār*	31-40 y	Never	Agree	Neither	45 c below D	50 c below E	Almost no deviation	(not provided)	10 c above F#/Gb
KM	*Santūr*	31-40 y	Sometimes	Agree	Agree	45 c below D	45 c below E	Almost no deviation	(not provided)	10 c above F#/Gb

The table is sorted in the ascending order of the years of experience (Q2); "Almost no deviation" was defined as a difference of ± 2 c.

APPENDIX 2: INDIVIDUAL PICTORIAL ANALYSIS RESULTS

Participant	C	D♭	D𝄳	D	E♭	E𝄳	E	F	F‡	F#	G𝄳	G	A♭	A𝄳	A	B♭	B𝄳	B	C'
Author	0	90	137	194	294	341	402	500	548	593	640	694	796	843	898	995	1044	1101	1200
AA1	0	N.A.	148	211	303	364	417	521	N.A.	602	665	722	815	872	916	1013	1062	1105	1200
TF	0	96	146	197	294	338	390	486	537	576	623	683	762	843	909	992	1048	1102	1200
BN	0	88	141	197	297	349	402	503	552	602	652	700	795	854	907	1005	1059	1105	1200
MM	0	104	149	202	294	351	400	492	N.A.	593	635	696	796	848	900	996	1051	1109	1200
PR	0	N.A.	137	199	288	336	394	493	561	N.A.	625	692	782	834	891	992	1047	1100	1200
AP	0	N.A.	129	192	290	335	393	494	550	613	N.A.	697	790	836	895	997	1046	1098	1200
HS	0	97	135	204	295	344	392	493	551	600	647	702	787	848	906	994	1057	1110	1200
AR	0	96	140	194	296	345	400	498	554	592	637	698	803	852	914	995	1052	1098	1200
AA2	0	N.A.	130	208	291	338	400	514	N.A.	590	637	714	796	852	909	990	1050	1112	1200
MP	0	100	148	200	280	342	397	487	542	590	642	693	778	843	903	985	1043	1091	1200

Interval sizes are in cents.