

Quantifying Alpatva (Scarcity) and Bahutva (Abundance): A Comparative Study of the Scale-identical Hindustānī Rāgas Bhūpālī and Deskār

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INTRODUCTION

IN Indian music, such as Hindustānī music—the predominant classical music or art music form of northern India, which this article is about—tonality is chiefly articulated through melody rather than through harmony (Castellano, Bharucha, and Krumhansl 1984; Clarke 2017). The various combinations of sharp and flat notes give rise to several basic scales each containing seven notes including the tonic and the fifth note, which are known as *thāt* in Hindustānī music, and from these heptatonic scales more complex musical frameworks are created.

[2] Like Karnāṭik music, its southern counterpart, Hindustānī music is based on the *rāga* system. Before going into the details, I wish to place the concept of *rāga* in the context of world music. A *rāga* is not merely a scale; it is a complex grammatical and aesthetic structure, similar to but with distinctive elements compared to the *maqām* in Arabian and Persian music or *dastgāh* in Persian music (Powers et al. 2001). In Western music, *modes* are created from a scale by treating different notes of the scale as the tonic note and thereby producing patterns of notes that correspond to other scales.¹ However, the creation of *rāgas* from *thāts* is an entirely different process from the creation of modes from scales, since in this case, the tonic note is not changed. One either uses all notes of the *thāt*, i.e., a heptatonic scale, or omits one or two notes (other than the tonic note) to create a pentatonic or hexatonic scale, and then introduces grammatical rules for performing the *rāga*, such as rules for which notes are to be used in the ascent and which in the descent, how each note is to be articulated—whether it is to be elongated or only sung fleetingly, whether it is colored by other notes, whether it is oscillated, and so on—and which combinations of notes are permissible and which are not. One can thus create

1. This process is also used in Hindustānī and Karnāṭik music, but to a much smaller extent. The Hindustānī music term for it is *mūrccanā*. It may be used for purposes of teaching or, occasionally, to create novelty in a performance. However, its role in Hindustānī and Karnāṭik music is not pivotal.

a large number of rāgas just from a single scale. I explain these rules in more detail in subsequent sections of this paper.

[3] This article deals with two modern scale-identical Hindustānī rāgas: Bhūpālī (also known as Bhūp), and Deskār. Both these rāgas have the notes (*svaras*) of the major pentatonic scale. The former rāga has existed since the 16th century, while the latter is much newer. This work aims to quantitatively describe and compare the rāgas and unearth features that are not explicitly known.

AIMS OF THE PRESENT STUDY

[4] While some elements of rāga grammar are explicitly described in the śāstra, other equally important features are learned only from the lakṣya, by listening to and imitating the guru or to fellow musicians. These seem to be subconsciously imbibed, and practitioners instinctively produce these features without being consciously aware of them. My study is aimed at discovering these implicit structures, which to the best of my knowledge have never been described in words before. My work shows that these too can be made explicit, and thus have the potential to be included in the śāstra. While it is easy to contrast rāgas that have different *svaras*, it is a lot harder to lay bare the differences in character between two rāgas that share a scale. I therefore choose the scale-identical pentatonic rāgas Bhūpālī and Deskār. In my study, I have devised several quantitative methods of differentiating between these rāgas. For the selected recordings identified in Table 2, I measure the following quantities: frequency of occurrence of pitches as a measure of the *svara*'s relative *alpatva* (scarcity) or *bahutva* (abundance), *svara* lengths (durations of individual occurrences of *svaras*) as a measure of *dīrgha bahutva* (one of the main kinds of *bahutva*, where a particular *svara* is often elongated) or its opposite, and *svara-saṅgati-s* (pairs of consecutively occurring *svaras*). I have also devised a clustering method to group similar renditions of a particular rāga together, based on ratios of occurrence frequency of other *svaras* to those of the *vādī* and *saṁvādī* (defined below).

THE SVARAS OF HINDUSTĀNĪ MUSIC

[5] Before going into the specifics of the two rāgas selected for this study, I would like to discuss the modern system of musical notes (*svara-s*) in Hindustānī music, and give some details of rāga structure. Table 1 lists the *svaras*.

Name	Short name	Modern Hindustānī name	Western scale degree	Western note-interval
1. Ṣaḍja This is not an absolute pitch, but is fixed at the pitch that is most comfortable for the voice or the instrument(s). The other svaras are then defined <i>relative</i> to it. It is called the “system tonic” (Powers et al. 2001; Qureshi et al. 2020).	Sa	Ṣaḍja (S)	1	Perfect unison
2. Ṛṣabha	Ri (Karnāṭik), Re (Hindustānī)	Komal ṛṣabh (r)	2, flat	Minor second
		Śuddha (tīvra) ṛṣabh (R)	2, sharp	Major second
3. Gāndhāra	Ga	Komal gāndhār (g)	3, flat	Minor third
		Śuddha (tīvra) gāndhār (G)	3, sharp	Major third

4. Madhyama	Ma	Śuddha (komal) madhyam (m)	4, flat	Perfect fourth
		Tīvra madhyam (M)	4, sharp	Augmented fourth
5. Pañcama	Pa	Pañcam (P)	5	Perfect fifth
6. Dhaivata	Dha	Komal dhaivat (d)	6, flat	Minor sixth
		Śuddha (tīvra) dhaivat (D)	6, sharp	Major sixth
7. Niṣāda	Ni	Komal niṣād (n)	7, flat	Minor seventh
		Śuddha (tīvra) niṣād (N)	7, sharp	Major seventh

Table 1. The twelve svaras of Hindustānī music (within any given octave).

[6] It is important to mention that the melody of the svaras as created by their distinctive rāga-specific ornamentations is more important in Indian music than the intervals are (Powers et al. 2001). The tonic note—a “system tonic” (Powers et al. 2001; Qureshi et al. 2020)—is called *ṣaḍja* (abbreviated to *sa*), and the other svaras are defined relative to it. Within one octave, there are twelve svaras including *sa*, each svara being one semitone higher than the previous. There are seven svara names: *ṣaḍja*, *ṛṣabha*, *gāndhāra*, *madhyama*, *pañcama*, *dhaivata* and *niṣāda*, respectively abbreviated to *sa*, *ri* (*re* in Hindustānī music), *ga*, *ma*, *pa*, *dha* and *ni*. Out of these, *re*, *ga*, *ma*, *dha* and *ni* have two variants each – one *komal* (flat), and the other *tīvra* (sharp), as shown in Table 1.² In this work, I represent the

2. Karnāṭik music defines the same svaras differently, but that is beyond the scope of this article.

former using lowercase letters, and the latter using uppercase letters. There is also the concept of *śuddha* (natural) and *vikṛta* (altered) notes. This is the one that is more commonly used in Hindustānī music today. In modern Hindustānī music, the *tīvra* variants of *re*, *ga*, *dha* and *ni* are considered *śuddha* svaras. In the case of *ma*, it is the komal variant (m) that is considered *śuddha*. *Sa* and *pa* are not given any qualifiers. R, G, m, D and N are known as *śuddha re*, *śuddha ga*, *śuddha ma* and so on. The remaining five svaras (r, g, M, d, n) are *vikṛta*, and their names are given the adjectives “komal” (r, g, d and n) and “*tīvra*” (M only). *Sa* and *pa* do not have any *vikṛta* variants, and are necessarily present in every *thāt* (pitch collection). A *thāt* is a collection of seven svara-pitches, *sa re ga ma pa dha ni*, and forms the basis for the creation of *rāgas*. Since there are only one *sa* and one *pa*, and two each, i.e., one *śuddha* and one *vikṛta*, in the other five categories (*re*, *ga*, *ma*, *dha* and *ni*), there are in theory $2^5 = 32$ possible *thāts*. The *śuddha thāt* of Hindustānī music corresponds to the major scale or Ionian mode in Western music. A *rāga* may take 5, 6 or all 7 svaras of its *thāt*. *Sa* being the tonic, it cannot be omitted from any *rāga*. Some *rāgas* admit both komal and *tīvra* variants of a particular svara. Unlike *thāts*, *rāgas* are not merely collections of pitches, but have a grammar, which is described in the next section.

[7] Once one fixes the tonic at a particular pitch (depending on the range of the voice or the instrument), there are three octaves, i.e., registers—*saptak*³ in Hindustānī music and *sthāyī* in Karnāṭik music—in which the singing or playing ranges. These are *mandra* (lower), *madhya* (middle) and *tāra* (higher). The *sa* of the *madhya saptak* is the tonic. *Mandra* svaras are represented by a dot below the svara symbol, e.g., Ṗ, and *tāra* svaras are represented by a dot above, e.g., Ṣ. A vocal performance typically ranges between Ṗ or ṁ/M and Ṣ. Some vocalists may go down to ḡ/G or ṛ/R, or even as low as Ṣ.

[8] There are numerous grammatical rules that make up a *rāga*. These are understood from both *lakṣaṇa* (what is prescribed) and *lakṣya* (what is seen in practice). It is usually taken for granted that musicians adhere to the grammar while singing or playing the *rāga*. *Rāga* grammar is typically learnt directly from a guru, or by attending or listening to lecture-demonstrations on the *rāga*, or, when one is a trained musician, by listening to multiple renditions of the *rāga*. Performed music constitutes the *lakṣya* (Bhatkhande 1910), which roughly translates to “what is observed.” It refers to the music as it is performed in the ‘present’ day. In the context of this paper, I choose to define the

3. The name *saptak* comes from *sapta*, meaning seven. This comes from the seven svaras, *sa re ga ma pa dha ni*. However, there are in reality twelve svaras in a *saptak*, since there are two each of *re*, *ga*, *ma*, *dha* and *ni*.

‘present’ day as beginning in the later part of the 20th century, i.e., the 1950s onwards, for two reasons. Firstly, recorded music, which now plays a major role in understanding rāgas, is only as old as the early 20th century in India. Also, a 6-volume series of music textbooks attributed to the musicologist Vishnu Narayan Bhatkhande⁴, now considered the standard, was published only in the 1930s, and Bhatkhande was alive till 1936. I thus treat the present age as beginning in the post-Bhatkhande era.

GRAMMAR OF RĀGAS

[9] Harold S. Powers has put forward a detailed theory of rāgas, comparing and contrasting them with maqāms, dastgāhs and Western modes (Powers et al. 2001). The term *rāga*⁵ in a musical sense first appeared in the 8th century (Qureshi et al. 2020), and rāga music has been evolving since then. Over the centuries, it has been passed down both orally from guru to disciple, but also textually, through various musical treatises.⁶

[10] The rules of rāga grammar are preserved in written form in the *saṅgīta śāstra-s* (musical treatises). Possibly the most important modern saṅgīta śāstra is the 6-volume *Hindustānī Saṅgīt Paddhati – Kramik Pustak Mālikā* (*The Hindustānī Music System – A Series of Books*) attributed to Vishnu Narayan Bhatkhande (1860–1936) (Bhatkhande 1920s–30s).⁷ Nearly every performer or scholar of Hindustānī music is familiar with this work.

4. Bhatkhande, Vishnu Narayan, 1920s–1930s. *Hindustānī Saṅgīt Paddhati – Kramik Pustak Mālikā*. For this paper, I have used the 1990 Hindi edition, published by Sangeet Karyalaya, Hathras.

5. The word has multiple meanings in Sanskrit, such as “color”, “dye”, “love”, “intense emotion.” Rāgas in music are characterized by the quality of *rañjana*, which literally means “the act of coloring or dyeing”, and is derived from the same root verb as *rāga* does. In a musical sense, *rañjana* refers to “coloring the mind”, i.e., being aesthetically pleasing.

6. Further details are provided in Appendix A.

7. Harishchandra Srivastava, editor of the Hindi edition of the *Kramik Pustak Mālikā*, in his preface to Vol. 3, titled *Kramik Pustak Mālikā kā Lekhak Kaun* (*Who is the writer of the Kramik Pustak Mālikā*), says: “...it appears that after Bhalchandra Sitaram Sukathankar, the previous publisher of this *Kramik Pustak Mālikā*, when this book reached the hands of another publisher, in order to make the books more important and useful, the author’s name was cited as Pt. Vishnu Narayan Bhatkhande, and all scholars continued to believe this to be true. Nobody tried to look in more detail. I too, following the view of the majority, am considering Paṇḍit Bhatkhande as the author. Here, I feel it necessary to remind [readers] that Paṇḍit Bhatkhande passed away in 1936, and during his lifetime, several editions of the second and third volumes of these books were published”. Whatever be the case, there is no doubt that the music theory in these books is Bhatkhande’s, going by his earlier works such as *Śrīmal-Lakṣyasaṅgītam* (Mumbai: Nirnaya-Sagar Press, 1910) and *Hindusthānī Saṅgīt Paddhati* (1910–1932, 4 volumes, distinct from the *Kramik Pustak Mālikā*, Hindi edition: Hathras: Sangeet Karyalaya, 4th edition 1974, reprint 2006). Also, as Srivastava mentions in his preface, notations of music compositions collected by Bhatkhande are present in the *Kramik Pustak Mālikā*. Since the 6-volume *Kramik Pustak Mālikā* has become the most important set of textbooks for modern Hindustānī music, I cite it at several points in this work. Following Srivastava, I too attribute it to Bhatkhande.

[11] The śāstra and lakṣya are both transmitted from guru to disciple, albeit in different ways. The śāstra is typically explained verbally, while the lakṣya is demonstrated by singing or playing an instrument. From the latter, the student learns the grammar in more implicit or subconscious ways, through hearing and reproducing what the guru sings or plays. Śāstra and lakṣya are not mutually exclusive, and they influence each other. Despite widespread knowledge of the śāstra, it is worth questioning to what extent it is consistent with lakṣya. Consistency between śāstra and lakṣya may vary from rāga to rāga.

[12] The *svarūpa* (aesthetic character, identity) of a rāga is composed of the following *lakṣaṇas* (characteristic features), which are outlined by the śāstra:

1. *Thāt*: According to Vishnu Narayan Bhatkhande, each rāga is mapped to a thāt (parent scale). Many rāgas, however, contain both the komal and the tīvra variant of one or more svaras. In such cases, the thāt is decided by which svara-variant is used to a greater extent as compared to the other. However, Bhūpālī and Deskār use only one variant (śuddha in this case) of each svara, and they are assigned thāts based on their similarities to other rāgas.
2. *Āroh*: Ascent, from S to S̄. This varies in the number of svaras present. It may be *auḍav* (pentatonic), *ṣāḍav* (hexatonic), or *sampūrṇa* (“complete,” i.e., heptatonic).
3. *Avaroh*: Descent, from S̄ to S. This may also be *auḍav*, *ṣāḍav* or *sampūrṇa*.
4. *Jāti*: Based on the number of svaras in the āroh and avaroh, rāgas may be classified as *auḍav-auḍav*, *ṣāḍav-ṣāḍav*, or *sampūrṇa-sampūrṇa* (equal numbers of svaras in āroh and avaroh). They may also be classified as *auḍav-ṣāḍav*, *auḍav-sampūrṇa* or *ṣāḍav-sampūrṇa* (unequal numbers of svaras in āroh and avaroh). Except for some rare exceptions, Hindustānī rāgas typically do not have less than five svaras, or less svaras in the avaroh than in the āroh. Bhūpālī and Deskār are both *auḍav-auḍav*, and their āroh and avaroh are SRGPDŚ-ŚDPGRS. *Auḍav* and *ṣāḍav* scales are known as “transilient scales” (Jairazbhoy 1971, 122), in the sense that they skip one or two svaras—e.g., skipping *ma* while moving from G to P or skipping *ni* while moving from D to Ś in Bhūpālī/Deskār.
5. *Vādī* and *saṁvādī*: Respectively the dominant svara of the rāga, and the svara that is consonant (in *saṁvād*)⁸ with it. Based on whether the vādī is in the *pūrvāṅg* (the region between S and P) or *uttarāṅg* (the region between m and S), rāgas may be

8. Saṁvād is defined as the occurrence of corresponding phrases with the same relative spacing of svaras in the pūrvāṅg and uttarāṅg (defined later in the paragraph) of a rāga, for example, the phrase RGPG in the pūrvāṅg of Bhūpālī, and its “mirror image” PDŚD in the uttarāṅg.

classified as *pūrvāṅ-pradhān* (pūrvāṅ-dominated) or *uttarāṅ-pradhān* (uttarāṅ-dominated) (Bhatkhande 1920s–30s vol.3, 15–16). The pūrvāṅ and uttarāṅ (Qureshi et al. 2020), while roughly corresponding to the lower and upper tetrachord in a sampūrṇa rāga, are not exactly the same as tetrachords, since *ma* and *pa*, the ‘midpoints’, are considered to belong to both. Thus, there are both pūrvāṅ- and uttarāṅ-pradhān rāgas with *ma* or *pa* for their vādī (Bhatkhande 1920s–30s vol.3, 16). The svaras other than the vādī and saṁvādī are known as *anuvādī*. The vādī of Bhūpālī is *ga* (Bhatkhande 1920s–30s vol.3, 23), and it is thus pūrvāṅ-pradhān (Bhatkhande 1920s–30s vol.4 p.247). On the other hand, Deskār is uttarāṅ-pradhān, because its vādī is *dha* (Bhatkhande 1920s–30s vol.4 p.247). The saṁvādī of Bhūpālī is *dha*, and that of Deskār is *ga* (Bhatkhande 1920s–30s vol.3, 23, 1920s–30s vol.4, 247).

6. *Samay*: The time of singing or playing (Yardi and Chew 2004). According to Bhatkhande’s theory, pūrvāṅ-pradhān rāgas have times ranging from noon to midnight, while uttarāṅ-pradhān rāgas have times ranging from midnight to noon. Bhūpālī is an early night rāga (Bhatkhande 1920s–30s vol.3, 23), and Deskār is a morning rāga (Bhatkhande 1920s–30s vol.4 p.247). While samay does not play a role in differentiating rāgas aesthetically, I mention it here because it is an important lakṣaṇa of Hindustānī rāgas.
7. *Nyās svaras*: The svaras on which one may end phrases. The act of ending a phrase on a particular svara is known as nyās.
8. *Varjit svaras*: Svaras whose usage is not permissible in a rāga. *Ma* and *ni* are varjit in both Bhūpālī and Deskār.
9. *Uccāraṇ*: the ‘pronunciation’ of each svara (Qureshi et al. 2020). For example, the phrase *ga pa dha sa* in Bhūpālī is rendered as ^PG P ^ṢD, ^ṢD, ^DṢ, where the superscripts are *kaṇ svaras*—short-duration particles of svaras used as an ornament on other svaras that are sung for a longer duration. The ^ṢD is rendered more forcefully and with a greater Ṣ-component in Deskār. Also, a plain, brisk GPDṢ is more characteristic of Deskār than of Bhūpālī.
10. *Alaṅkāra*: These are ornamentations used on svaras (Qureshi et al. 2020). Examples are *mīṇḍ* (glissando), *āndolan* (gentle oscillation of a svara using kaṇas of other svaras), *gamak* (a momentary push of forceful singing from the chest), etc. Some rāgas by nature use a great deal of *mīṇḍ*, but this is not true for Bhūpālī and Deskār. Kaṇas and alaṅkāras influence the precise *śruti* (microtone) of a svara.

11. *Rāgāṅga* or *aṅga*: *Aṅga* literally means limb or part. In the context of rāgas, it refers to characteristic phrases, complete with rāga-specific uccāraṇ, that give a rāga its identity. Sometimes several rāgāṅgas may be present in one rāga.
12. *Calan*:⁹ The word literally means ‘movement’. It refers to the overall structure of the rāga. The inventory of phrases, and the general pathways that one can follow while improvising: which phrase(s) can follow which phrase(s), and so on. This is somewhat like syntax in the grammar of a language. The lakṣaṇas defined in this list are basic components of the calan, but do not constitute its entirety.
13. *Alpatva* and *bahutva*: See below.

ALPATVA AND BAHUTVA

[13] *Alpatva* and *bahutva* are the primary focus of this article. These refer respectively to the scarcity and abundance of svaras in a raga. This scarcity or abundance is of many types, which are described below.

[14] **Bahutva** (Bhatkhande 1920s–30s vol.4, 34; Widdess 1995, 47): literally speaking, the state of a svara being “more” in comparison to other svaras of the rāga. The vādī is thus the svara with the greatest bahutva. Bahutva is of the following types:

1. *Nyās bahutva* – A kind of abundance generated through the greater likelihood of a svara appearing as a *nyās* point (defined above), given the function of *nyās* svaras in ending phrases. The tonic note, *sa*, is by definition a *nyās* svara in every rāga. Therefore, *nyās* on *sa* is not considered special. Performers or scholars of Hindustānī music talk about svaras other than *sa* that are *nyās* points. In Bhūpālī, the *nyās* svara (other than *sa*) is *ga* alone. *Re*, *pa*, and *dha* can be points of what I call ‘ardha *nyās*’ (half *nyās*). If *nyās* is a full stop, then ardha *nyās* is a comma. A *nyās* on *dha* or *pa* would cause a shift into Deskār, and a *nyās* on *re* would cause an entry into the territory of Rāga Śuddha Kalyāṇ. The Karnāṭik rāga Mohanam, which is *ga*-dominated like Bhūpālī, has *nyās* on all these svaras.
2. *Dirgha bahutva* – elongation of a svara, as demanded by the structure of the rāga. *Ga* and *pa* have *dirgha bahutva* in Bhūpālī, and *dha* has *dirgha bahutva* in Deskār.

9. The letter *c*, when used in Indian words in this paper, represents the unvoiced alveolo-palatal affricate, च in the Devanāgarī script.

3. *Abhyās bahutva* – a combination of *dirgha* and *nyās bahutva*, i.e. repeated elongation of and *nyās* on a *svara*. *Ga* has *abhyās bahutva* in *Bhūpālī*, and *dha* and *pa* in *Deskār*.
4. *A-laṅghan bahutva* – When there is no *laṅghan* (skipping over) of a *svara*, because the structure of the *rāga* does not allow it. *Laṅghan* will be defined in the next section.

[15] **Alpatva** (Bhatkhande 1920s–30s vol.4, 33; Widdess 1995, 47): the state of a *svara* being *alpa* (‘less’ or ‘reduced’) in comparison to other *svaras* of the *rāga*. It is of the following types:

1. *Anabhyās alpatva* – an absence of *dirgha* and *nyās bahutva*, i.e., the scarcity of a *svara* because it cannot function as a phrase-ending (*nyās*) and is not elongated. The opposite of *abhyās bahutva*.
2. *Laṅghan alpatva* – Skipping over of a *svara*. This is not the same as the state of being *varjya* (completely absent from the *rāga*). The *svaras ma* and *ni* are *varjya svaras* in both *Bhūpālī* and *Deskār*. We thus cannot say that there is *laṅghan* of these *svaras*, since they are altogether absent. *Laṅghan* typically happens when a *svara* that *is* present in the *rāga* is already *alpa*, usually in the sense of appearing only as a passing note and never being elongated, and can therefore be sometimes skipped. The *svara re* undergoes *laṅghan* in the *āroh* (ascent) in some renditions of *Deskār*.

[16] *Rāgas* whose *thāṭṣ* differ can easily be told apart. Within one *thāṭ*, differences in *āroh* and *avaroh* serve to differentiate between *rāgas*. However, there are also several sets of *rāgas* possessing identical *āroh* and *avaroh*, which I term scale-identical *rāgas*, and these cannot be differentiated based on *svaras* alone. One such set is *Bhūpālī*, *Deskār*, *Jait Kalyāṇ*, and the *auḍav* variety of the *rāga Śuddha Kalyāṇ*, all of which have the major pentatonic scale, S R G P D (all *tīvra*, or all *śuddha*) (Bhatkhande 1920s–30s vol.3, 23). This article deals only with the first two, since they are more popular. When *rāgas* are scale-identical, they can be differentiated based on the following *lakṣaṇas* –

- Whether a *raga* is *pūrvāṅg-* or *uttarāṅg-pradhān* (upper or lower tetrachord dominant). For example, *Bhūpālī* is *pūrvāṅg-pradhān*, while *Deskār* is *uttarāṅg-pradhān*.
- *Vādī-saṁvādī*. For example, the *vādī-saṁvādī* of *Bhūpālī* are G and D, and vice versa in *Deskār*.

- Rāga-specific uccāraṇ of svaras or phrases. For example, the phrase *sa re dha sa* is rendered as SR, ^SD, ^SD, ^DS in Bhūpālī, and as a plain SRDS in Deskār.
- Alpatva and bahutva of svaras other than the vādī. For example, R is much weaker in Deskār than in Bhūpālī.
- Characteristic phrases that make up the calan. For example, GDPG is a phrase of Bhūpālī, while PDGP is a phrase of Deskār.

[17] Using the same svaras, a performer can create entirely different aesthetic atmospheres by varying the vādī-saṁvādī, alpatva and bahutva, uccāraṇ or overall calan. Changing any of these can take one into a different rāga. This is why the study of scale-identical rāgas is important.

IMPORTANT LAKṢAṆAS OF BHŪPĀLĪ AND DESKĀR

[18] Bhūpālī: One of the most popular Hindustānī rāgas (S. Rao et al.).

- Āroh-avaroh: S R G P D Ś – Ś D P G R S (corresponding to the major pentatonic scale in Western music, and to the Karnāṭik raga Mohanam).
- Thāṭ – Kalyāṇ (SRGMPDN, all svaras tīvra). Since *ma* and *ni* are absent, Bhūpālī could be derived from any out of four different thāṭs. However, owing to its closeness to the rāga Śuddha Kalyāṇ, whose avaroh is sampūrṇa (heptatonic), it is placed in the Kalyāṇ thāṭ (Bhatkhande 1920s–30s vol.3, 23).
- Traditional time of singing: Early night, along with other rāgas that possess the Kalyāṇ rāgāṅga.
- Vādī – *ga*, saṁvādī – *dha* (Bhatkhande 1920s–30s vol.3, 23). Since the vādī is in the pūrvāṅg, Bhūpālī is considered pūrvāṅg-pradhān.
- Nyās svaras other than *sa*: Primarily *ga* in both the āroh as well as avaroh, although an *avarohātmak* (descending) nyās on *dha* may occur.
- Characteristic *vakra* phrases (phrases where the svaras are randomly ordered): SRPG, GRPG, GDPG, SR^SD^SDS .
- Alpa svaras: None.
- Low density of mīṇḍ.
- In Bhūpālī, there is typically no laṅghan of any svara. Thus, all the svaras have *alaṅghan* bahutva.

[19] Deskār: While less commonly performed than Bhupālī, this rāga is familiar to most Hindustānī musicians owing to its closeness to Bhupālī (S. Rao et al.).

- Āroḥ-avaroh: S R G P D Ś – Ś D P G R S (same scale as Bhūpālī, but different treatment of the svaras).
- Thāt – Bilāval (SRGmPDN, all svaras śuddha) (Bhatkhande 1920s–30s vol.4, 247). This is due to Deskār’s uttarāṅg-pradhān nature and its closeness to the heptatonic uttarāṅg-pradhān rāga Bilāval.
- Traditional time of singing: Morning, along with other rāgas that possess the Bilāval rāgāṅga.
- Vādī – *dha*, saṁvādī – *ga*. Since the vādī is in the uttarāṅg, Deskār is considered uttarāṅg-pradhān.
- Nyās svaras other than *sa*: *Dha* in the āroḥ, *pa* in both āroḥ and avaroh.
- Characteristic vakra phrases: SRDŚ, PDGP, D_Ś^{DP} (where the underscore indicates lengthening). Other phrases that set Deskār apart from Bhūpālī are the S-D jump and the plain GPDŚ without the kaṇ of Ś in D.¹⁰
- Alpa svara: R. It has anabhyās alpatva, i.e., R is never dīrgha, and there is no nyās on it. In the avaroh, it is just a transitory note between *ga* and *sa*, appearing as a *kaṇa* (particle) svara, G^R S. In many people’s renditions, it undergoes laṅghana in the āroḥ, with SGP being sung instead of SRGP (where there is nyās on *pa*). But R may still be present in the phrases SRGPD and SRGP^SD (with nyās on *dha*). There may be PDŚRĠRŚ or PDŚRĠ^RŚ in the tāra saptak. In Kishori Amonkar’s version, the combination SRG almost never appears – not even in the tāra saptak, excepting for a slight hint occasionally – and only SG is present for most of the performance. However, in the approach of my guru, Professor Ojesh Pratap Singh, SGP is considered a phrase of the rāga Jait Kalyāṇ¹¹, and Deskār must take SRGP. This is something my guru told me while teaching me Deskār. There is then no laṅghana of *re* in Deskār in his style. It is alpa only in the sense of anabhyās.

HOW A RĀGA IS PERFORMED

[20] Hindustānī rāga music has three main genres within it – *dhrupad*, *khayāl* and *thumrī*. Dhrupad and khayāl are *rāga-pradhān*, dominated by the rāga. While they sound very different, they have several common features. Instrumental music often combines

10. The former phrase typically makes it immediately obvious to a trained ear that the rāga being heard is Deskār. While the latter occurs in Bhūpālī in fast-paced segments, it is characteristic of Deskār in the slower segments, which form the bulk of the performance of a rāga. In the slow part of a performance of Bhūpālī, D in the phrase *ga pa dha sa* is always coloured with a kaṇ of Ś.

11. Another rāga with the major pentatonic scale.

elements of two or all three of these systems. Some features that set *ḵhayāl* apart from *dhrupad* are a greater variety of *alaṅkārs*, and a higher possibility of undulating and fast-paced phrases that only involve the sound *ā*. *Dhrupad* typically has only *gamak*, *mīṇḍ* and *āṇḍōlan*. *Ḷhayāl* has these, but also additional *alaṅkārs* such as *ḵhaṭkā* and *murkī* that pack several *svaras* into a minute length of time. *Ḷhayāl* also has more intra-phrase tempo variation in the *ālāp* section (exposition of *rāga* phrases, described below) than *dhrupad* does, and some additional features mentioned in the following paragraphs.

[21] *Rāga* performance is typically structured as follows (Powers et al. 2001; Qureshi et al. 2020): *ālāp* (slow exposition of the pitches and phrases of the *rāga*), followed by *bandīś* (composition). A *bandīś* has a *tāla* (rhythmic cycle) and *laya* (tempo). The *laya* can be *vilambit* (slow), *madhya* (medium) or *drut* (fast). The general pattern is to perform a *vilambit* or *madhya-laya bandīś* first, and follow it up with a *drut bandīś* in the same *rāga*. The performance of the composition, though it begins with the *bandīś* as it has been composed or learnt, deviates from the pre-defined structure, and several varieties of on-the-spot improvisations are performed, such as *bol-ālāp* (*ālāp* using the words of the *bandīś*), *layakāri* (rhythmic variations using the words of the *bandīś*, either at the same pace as the *bandīś* or at faster paces), *sargam* (singing the *svaras* as sol-fa syllables), and *tān* (fast cascades of musical notes sung using the vowel *ā*). *Layakāri* is found in both *dhrupad* and *ḵhayāl*. *Bol-ālāp*, *sargam* and *tān* are typically found in *ḵhayāl* but not in *dhrupad*. The *ālāp* is an impromptu exploration of the various phrases possible in a *rāga*. It and other improvisations must be original, innovative and mostly non-repetitive, but never deviating from *rāga* grammar. In fact, multiple performances of the same *rāga* by the same musician, since they are improvised, sound different from one another, but owing to the adherence to grammar, it is still recognizable as the same *rāga*. The improvisation is not entirely impromptu, since a great deal of planning (Clarke 2017) goes into it when the musician is off-stage. Some musicians—examples being Kishori Amonkar and Kumar Gandharva—intentionally and cleverly deviate from the grammar, but still maintain the aesthetic atmosphere of the *rāga*, and this is a skill that is valued.

[22] In commonly heard styles of *dhrupad*, *ālāp* is typically performed separately, before the composition, and is a long piece. In *ḵhayāl*, though there is a short initial *ālāp* preceding the composition, the bulk of the *ālāp* is performed as part of the improvisation in the *vilambit* composition, though some *dhrupad*-influenced *ḵhayāl gharānā-s*¹² (loosely

12. Traditionally, these were families of hereditary musicians attached to different royal courts, each family having its own style of rendering *rāgas*. They also trained students outside the family in their style. *Gharānās*

translated as ‘schools of music and musical thought’) may perform longer, separate ālāps. The style of the Agra gharānā is an example of this. A *khayāl* ālāp may use specific syllables like those used in dhrupad ālāps, or may just use the vowel *ā*. Some gharānās—e.g., the Jaipur-Atrauli gharānā¹³, in whose style Mallikarjun Mansur, one of the artistes in this study, sings—may perform little to no initial ālāp. For a pūrvāṅg-pradhān rāga like Bhūpālī, the ālāp usually begins by establishing *sa*, spending some time in the mandra saptak, and gradually singing phrases that introduce successively higher svaras, in the order of the svaras (e.g., R, G, P...), entering the tāra saptak, going up to the tāra pañcam, and eventually coming back to *sa*. The ālāp does not merely ascend to the highest point and then descend. It tends to be rendered in segments that are punctuated by returning to the tonic (Qureshi et al. 2020). Modern ālāp-s typically have many segments, presided over by successively higher svaras. Uttarāṅg-pradhān rāgas like Deskār introduce the higher svaras much earlier, and might jump to the tāra *sa* quite early in the ālāp. The calan of the rāga is determined by the location of the vādī svara. Thus, uttarāṅg-pradhān rāgas are often performed in a more phrase-based manner, while pūrvāṅg-pradhān rāgas are performed in a svara-based fashion. Longer ālāps are possible in rāgas of the latter category, such as Bhūpālī.

[23] The initial part of the pre-bandīś ālāp is always slow in tempo and without an obvious rhythm, but it may be followed by faster segments that are rhythmic. This is especially true for dhrupad, where after singing a slow, free-flowing vilambit ālāp, there is a rhythmic madhya ālāp. Instrumentalists call this *joḍ*. After this segment, there is a much faster rhythmic segment, the drut ālāp (*jbālā* in instrumental terms). The *joḍ* and *jbālā* are played by instrumentalists even if their overall style is more *khayāl*-like (in terms of alaṅkārs, as described in the first paragraph of this section). These faster ālāp segments may sometimes also be performed by *khayāl* vocalists, typically in the Agra gharānā.

[24] After singing the bandīś as it is composed, the musician improvizes. In this segment, dhrupad musicians typically only perform layakārī (of different varieties). In *khayāl*, after sufficient exploration of the ālāp segment of the vilambit composition, the tempo is

are typically named after the capital cities of the courts to which they were attached. Today, gharānās survive in the form of distinct musical styles that are performed and imparted to students. However, in modern times, there is a great deal of intermixing of gharānā styles, especially in *khayāl*. Many musicians—for example, Kishori Amonkar and Kumar Gandharva—have created individual styles that cannot be mapped to any one gharānā style.

13. Kishori Amonkar too received training in the style of this gharānā, but went on to create her own style that was unbounded by gharānā.

typically increased, and layakārī, sargam and t̄an are performed. T̄an is a feature of k̄hayāl alone, and not of dhruḍpad. A k̄hayāl ālāp differs from a dhruḍpad ālāp in one more sense – the use of slower t̄an-like fragments in the ālāp itself.

[25] The vocal recordings I have used for this study are all k̄hayāl. The instrumental recordings, too, are closer to k̄hayāl than to dhruḍpad, though some of them contain dhruḍpad-like features (ālāp, joḍ and jhālā, or ālāp and joḍ).

[26] All the vocal recordings in this study contain melodic as well as percussive (tablā) accompaniment, and all the instrumental recordings—except two flute recordings that only consist of ālāp, joḍ and jhālā—contain tablā accompaniment in the bandīś section. In Hindustānī music, melodic accompanists are trained to follow the style and rāga interpretation of the main performer. Thus, my study makes the fair assumption that the rāga svarūpa played by the melodic accompanist does not differ greatly from that sung by the main vocal artiste.

DISCOGRAPHY

[27] The recordings of Bhūpālī (vocal and instrumental) and Deskār (vocal) used for this study are described in the discography in Table 2 below. The choice of artistes was restricted to well-known and established musicians, and also biased by availability. To present a more comprehensive view of how rāgas are performed, I also included artistes such as Kishori Amonkar and Kumar Gandharva, who deviate from gharānā norms. Complete recordings were used in all cases.

[28] I have used the following recordings for this study:

Bhūpālī (n=14, 7/14 vocal)

Artist	Instrumentation	Duration (min)	Album details
Kishori Amonkar	Vocal	21:00 (out of which 15:09 constitute the vilambit composition, and the rest the drut composition. No initial ālāp)	Kishori Amonkar (vocal), <i>Best of Kishori Amonkar, Vol. 1</i> , Myuzic Entertainment, 2018

Kumar Gandharva	Vocal	Madhya laya: 9:58 (Initial ālāp up to 0:27)	Kumar Gandharva (vocal), <i>The Last Word in Hindustani Vocal, Pt. I</i> , Music Today, 2009
Prabhakar Karekar	Vocal	Vilambit: 30:11 (Initial ālāp up to 2:47)	Prabhakar Karekar (vocal), <i>Faces Of Raga—Prabhakar Karekar</i> , Living Media India Ltd., 1996
Mashkoo Ali Khan	Vocal	Drut tarānā: 4:57 (No initial ālāp)	Mashkoo Ali Khan (vocal), <i>Transcendence</i> , Nimbus Alliance, 2017
Rashid Khan	Vocal	22:20 (Initial ālāp up to 0:38, madhya laya composition up to 15:11, followed by drut)	Rashid Khan (vocal), <i>Masterworks from the NCPA Archives: Rashid Khan (Remastered)</i> , recorded 1984, NCPA under exclusive license to Sony Music Entertainment India Pvt. Ltd., 2011
Padmini Rao	Vocal	28:07 (Initial ālāp up to 2:55, vilambit up to 18:58, followed by drut)	Padmini Rao (vocal), <i>Aananda</i> , Super Cassettes Industries Private Limited, 2014
Padma Talwalkar	Vocal	16:24 (initial ālāp up to 3:55, then slow madhya laya tarānā)	Padma Talwalkar (vocal), <i>Tarana—Flights of Melody</i> , Original Sound Recording, 1994

Hariprasad Chaurasia	Bāṣurī (flute)	Ālāp, joḍ and jhālā: 32:12 (no gat in this recording)	Hariprasad Chaurasia (bāṣurī), <i>From the NCPA Archives—Hariprasad Chaurasia (Remastered)</i> recorded 1984, NCPA under exclusive license to Sony Music Entertainment India Pvt. Ltd., 2011
Rakesh Chaurasia	Bāṣurī	Ālāp: 12:14 Joḍ: 13:28 Madhya laya gat (composition): 22:54 Drut gat: 22:13	Rakesh Chaurasia (bāṣurī), <i>Call of Krishna</i> , Sona Rupa Ltd., 2004
Pravin Godkhindi	Bāṣurī	Joḍ, jhālā: 22:00	Pravin Godkhindi (bāṣurī), <i>Antaryami: Classical Flute Music for Relaxation & Meditation</i> , Sagar Music, 2005
Bismillah Khan	Śahnā'ī (wind)	16:49 (ālāp up to 1:50, followed by madhyalaya, speeds up from 9:28 onwards)	Bismillah Khan (śahnā'ī), <i>Evening Raga</i> , Classic Music, 2019 (obviously recorded much earlier, since the artiste passed away in 2006)
Sultan Khan	Sāraṅgī (string, bowed)	53:15 (ālāp up to 17:40, then vilambit, then drut from 42:46)	Sultan Khan (sāraṅgī), <i>Sultan Khan & Zakir Hussain</i> , Moment Records, 1992

Lalmani Misra	Vicitra Vīṇā (string, fretless. Played by plucking at one end, and at the other end, varying string length by sliding a rounded glass object on it)	12:53 (ālāp up to 2:59, then madhya laya composition)	Lalmani Misra (vicitra vīṇā), <i>Heritage Alive, Vol. 1</i> , recorded 1970, CD Baby 2013
Gianni Ricchizzi	Vicitra Vīṇā	Ālāp:12:05 Jod: 6:28	Gianni Ricchizzi (vicitra vīṇā), <i>Digital Booklet— Raga Bhupali & Raga Yaman—Anna Maria Mucilli, Gianni Ricchizzi & Giuseppe Fiore,</i> Wyastone Estate Limited, 1994

Deskār (all vocal; n=8)

Artist	Instrumentation	Duration (min)	Album details
Kishori Amonkar		29:55 (initial ālāp up to 0:58, followed by vilambit, followed by drut at 22:23)	Kishori Amonkar (vocal), <i>Best of Kishori Amonkar,</i> <i>Vol. 1</i> , Myuzic Entertainment, 2018 (The Bhūpālī and Deskār recordings are part of the same album, but were most probably sung on different occasions)
Arati Ankalikar-Tikekar		17:13	Arati Ankalikar-Tikekar

	(initial ālāp up to 1:45, followed by drut, then a faster drut tarānā at 12:45)	(vocal), <i>Classical Vocal: Aarti Ankalikar-Tikekar</i> , Fountain Music Company, 1997
Ajoy Chakraborty	21:29 (Initial ālāp up to 0:45, followed by slow madhyalaya composition, followed by drut at 14:21)	Ajoy Chakraborty (vocal), <i>Gharana Series—Patiala</i> , Living Media India Ltd., 1995
Kumar Gandharva	11:01 (madhyalaya composition) No initial ālāp	Kumar Gandharva (vocal), <i>Classical Vocal: Pt. Kumar Gandharva</i> , Fountain Music Company, 1999
Bhimsen Joshi	18:05 (Initial ālāp till 3:01, followed by drut)	Bhimsen Joshi (vocal), <i>Ragas Deshkar, Hindol, Jogia, Bhairavi</i> , Navras Records, 1994
Archana Kanhere	9:56 (Initial ālāp till 0:28, followed by drut, followed by drut tarānā at 5:36, slightly faster than previous composition)	Archana Kanhere (vocal), <i>Bandish: Raja Kale & Archana Kanhere</i> , Fountain Music Company, 2012 (Only Archana Kanhere part analysed)
Rashid Khan	21:55	Rashid Khan (vocal), <i>The Song Of Shiva</i> , recorded

	(Initial ālāp till 0:26, followed by drut)	1995, Navras Records, 1998
Mallikarjun Mansur	20:21 Negligible initial ālāp, vilambit-madhyalaya composition starts at 0:14, no drut	Mallikarjun Mansur (vocal), <i>Morning & Evening Ragas</i> , The Indian Record Mfg. Co. Ltd., 1990

Table 2. The recordings used in this study.

SOME OTHER STUDIES ON QUANTIFYING RĀGA STRUCTURE

[29] I now discuss earlier literature in the area of quantification of rāga structure. I do not list these exhaustively, but provide some examples of significant research in the field. An early study towards quantifying rāga structure was carried out in 1977 by the Karnāṭik musician-scholar T. Viswanathan (1977). This contained a study of alpatva and bahutva, albeit in a Karnāṭik context. Viswanathan studied ālāpanās¹⁴ (i.e. ālāps) of major heptatonic Karnāṭik rāgas of different melas—Bhairavi, Kalyāṇi, Śaṅkarābharaṇam, Toḍi, Sāveri and Kāmbhoji¹⁵—by many artistes, and made detailed plots that depict the relative frequencies of occurrence of individual svaras in each rāga ālāpanā. He also studied the usage of gamakas (the Karnāṭik term equivalent to alaṅkārs) of different kinds, characteristic identifying phrases of the rāgas, special rāga phrases, and many other details of the ālāpanā. In the present study, I have done a similar analysis to Viswanathan’s study of relative occurrence of svaras, the difference being that rather than studying rāgas from diverse scales, I have compared two scale-identical pentatonic Hindustānī rāgas, Bhūpālī and Deskār, and have also compared vocal recordings of Bhūpālī to instrumental ones. Another point of difference is that my study is not limited to the ālāp, and also takes into account the bandiś component. Of course, unlike Karnāṭik music, the performance of kḥayāl music or non-dhrupad instrumental Hindustānī music contains ālāp not just in the beginning but also within the bandiś. Also, I have analysed other quantitative metrics: svāra-saṅgatis, occurrence of other svaras relative to the vādī and saṁvādī, and svāra

14. Ālāp is known as ālāpanā in Karnāṭik music.

15. These rāgas are overall heptatonic. The first four are sampūrṇa-sampūrṇa, the fifth is auḍav-sampūrṇa, and the last is śāḍav-sampūrṇa.

lengths. I have used k-means analysis to separate vocal renditions of Bhūpālī from those of Deskār, and to separate vocal renditions of Bhūpālī from instrumental ones.

[30] Another study focused on quantification of svara frequency is the 1984 study by Castellano, Bharucha, and Krumhansl (1984), where listeners were asked to give ratings to svaras in different rāgas—the ten rāgas that lend their names to Bhatkhande’s ten thāṭs. The listeners were first subjected to a musical piece, following which they were made to listen to “probe tones” (single pitches, one of the twelve svara-pitches). They had to rate these probe tones in terms of how well these tones corresponded musically with the musical piece they had just heard. The highest ratings were given to *sa* and *pa* (possibly owing to the tanpura drone). The next highest rating was given to the vādī. The ratings resulted in plots of “tonal hierarchy” of svaras for these rāgas. My study analyses such parameters for Bhūpālī and Deskār, though based on exact measures obtained quantitatively through a computational study of recorded music, rather than being based on the perception of listeners.

[31] The Automated Transcription Project for Indian Music (AUTRIM) (S. Rao et al.), a joint project of the National Centre for Performing Arts (NCPA), India, and the University of Amsterdam (UvA), is an excellent resource for visualizing the svaras of a rāga as they are being sung. It makes use of the Praat software developed at UvA to convert rāga performances into detailed plots that can be visualized as “Music in Motion”. It contains such data for performances of more than 80 rāgas, Bhūpālī and Deskār included.

[32] David Clarke’s (2017) study on one ālāp in the rāga Yaman attempts to apply theories from Lehrdahl and Jackendoff’s 1983 work “A Generative Theory of Tonal Music” (GTTM) to Hindustānī music. Clarke finds that though GTTM has some limitations when applied to Indian music, at least a few features of Hindustānī music—such as the structuring of an ālāp that I described in a previous section—can be modelled using it.

[33] Preeti Rao and others have done far-reaching quantitative studies on rāga music. Some examples of the topics they cover are similarity between rāgas as understood from bandīś notation (Ross et al. 2017), classification of characteristic rāga phrases (P. Rao et al. 2014), discernment of these phrases by trained musicians (Ganguli and Rao 2019), identifying these phrases in audio recordings (Ross and Rao 2012), and differentiating between Hindustānī, Karnāṭik and Turkish music based on melodic features (Vidwans, Verma, and Rao 2020).

[34] To the best of my knowledge, there have been very few quantitative studies of alpatva and bahutva. I have already given the examples of the studies by Viswanathan and by Castellano *et al.* An important recent study by Ganguli and Rao compares rāgas with the same scale on the basis of tonal hierarchy, here based on the occurrence frequency and duration of svaras (Ganguli and Rao 2018). The present study has a different focus, since I have defined new quantitative measures.

METHODS

[35] The Parselmouth code used to extract data from the recordings requires them to be converted into mp3 files. Movavi Video Converter 20 Premium was used for this purpose.¹⁶

[36] To analyze the svara content of the recordings, a modified version of Parselmouth code (the Python version of UvA's Praat software) was used (Jadoul 2020), specifically the `draw_pitch` function. Each point in the extracted data represents the dominant frequency in 10 milliseconds, so there are 100 data points per second. The data thus obtained was plotted on Excel. Due to the drone of the tānpurā, the tonic note *sa* appears as a thick horizontal line in the plot. From its position, the pitch value corresponding to *sa* was ascertained by eye (see Figure 1). For each recording, the pitch values were normalized to the *sa* pitch.

16. <https://www.movavi.com/videoconvertermac/>

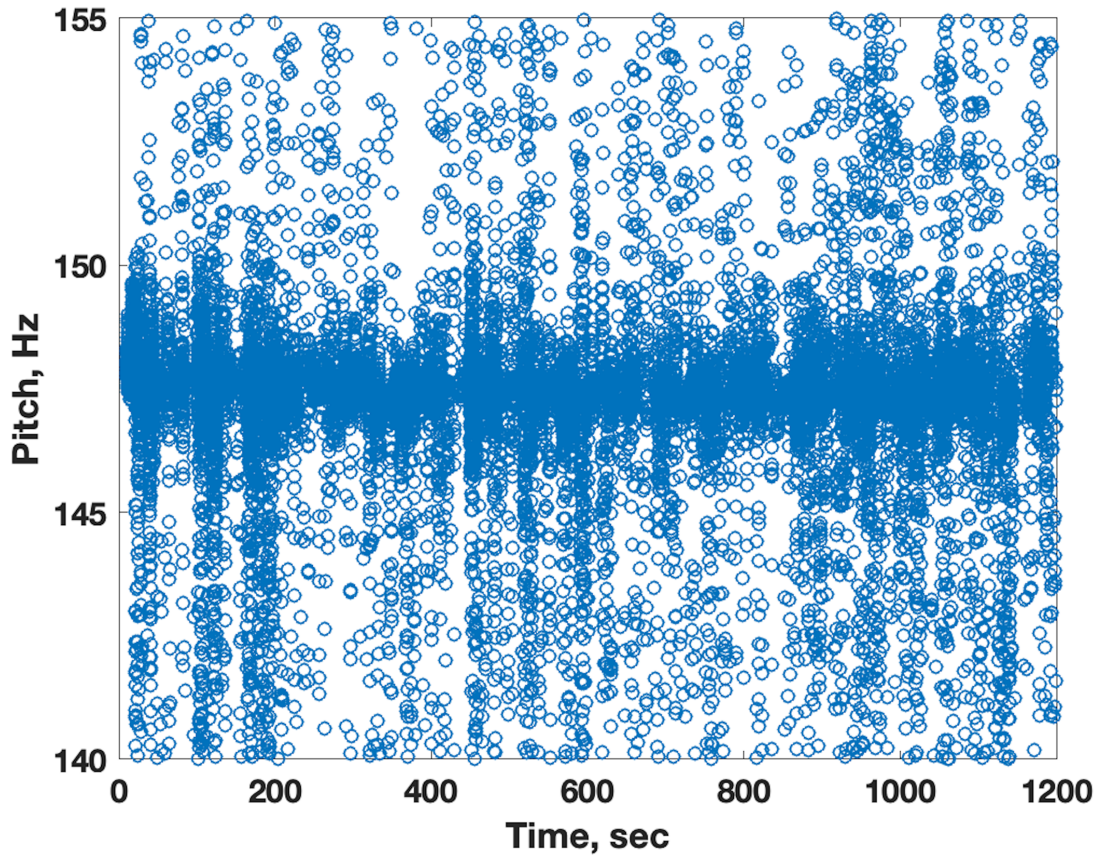


Figure 1. Determination of *sa* pitch using the pitch values extracted by Parselmouth—zoomed-in view of the relevant pitch range. The tānpurā drone is visible as a dense collection of data points.

This is part of the plot obtained from the recording of Deskār by Ajoy Chakraborty, and the *sa* pitch was determined as 147.4 Hz.

[37] After this, FORTRAN was used to bin the data according to svara frequency, and to select only those frequencies that apply to svaras of the rāga. In a procedure similar to that described in Rao *et al.* (P. Rao *et al.* 2014), all frequencies to within 50 cents of a given svara were assigned to that svara (except in the data presented in Figures 3 and 4). Each svara was then assigned a number, starting with 1 for \dot{P} , the *pa* of the mandra octave. For convenience, only the ‘major pentatonic’ svaras were assigned numbers. *Ma* and *ni*—whether komal or *tīvra*—and komal variants of *re*, *ga* and *dha* were not considered, since they do not appear in the rāgas of the present study. Svaras higher than \dot{P} (*tāra pa*) were also not considered, since their high pitch values mean that they are used extremely rarely, only in displays of exceptional virtuosity, such as in Sultan Khan’s recording of Bhūpālī, where the *ati-tāra sa* appears several times. Even in *uttarāṅg-pradhān* rāgas, svaras higher than \dot{P} are not considered essential.

[38] Madhya *sa* (S) and mandra *pa* (Ṙ) have also been excluded from the analysis, since they do not bear much on the differences between the two rāgas, and it is in the usage of every other svara that the main differences between Bhūpālī and Deskār lie. Moreover, the tānpurā produces a constant drone of S and Ṙ. The Parselmouth code cannot distinguish the tānpurā sound from the voice or main instrument, and as a result, S and Ṙ have high values in the probability distribution plots (Figures 2 and 3). I then attempted to reduce the tānpurā sound from the recordings using different parameters. This produces relatively minor quantitative differences in the data, but does not qualitatively change any of the conclusions of the study as compared to no reduction. Thus, the data for this manuscript was obtained without tānpurā reduction. This is discussed in greater detail in Appendix B.

[39] Using MATLAB, the percentage distribution of appearance frequencies (how often each svara appears) was plotted. Transition matrices (Bhattacharjee and Srinivasan 2011) were also prepared, which show the distribution of svara pairs, since svara-saṅgatis are an important component of rāga structure. The code treats only a pair of two non-identical svaras as a saṅgati. This means that SS, RR, GG and so on are not considered. Therefore, the main diagonal of the transition matrix appears as zeroes. This will become clearer in the next section.

RESULTS

1. PROBABILITY DISTRIBUTION OF SVARAS

[40] The distribution of occurrence of all frequencies from the data acquired from all artistes is shown in Figure 2 and 4 (Bhūpālī vocal vs instrumental), and figure 3 and 5 (Bhūpālī vocal, 7 samples, vs Deskār vocal, 8 samples). I have often felt that instrumentalists' approaches to rāgas may differ from those of vocalists, and this is why I compared vocal and instrumental renditions of Bhūpālī. As for Deskār, instrumental recordings are less common, and thus, I only compared vocal recordings of Deskār to vocal recordings of Bhūpālī. Here each data point was slotted into one of 500 frequency bins.

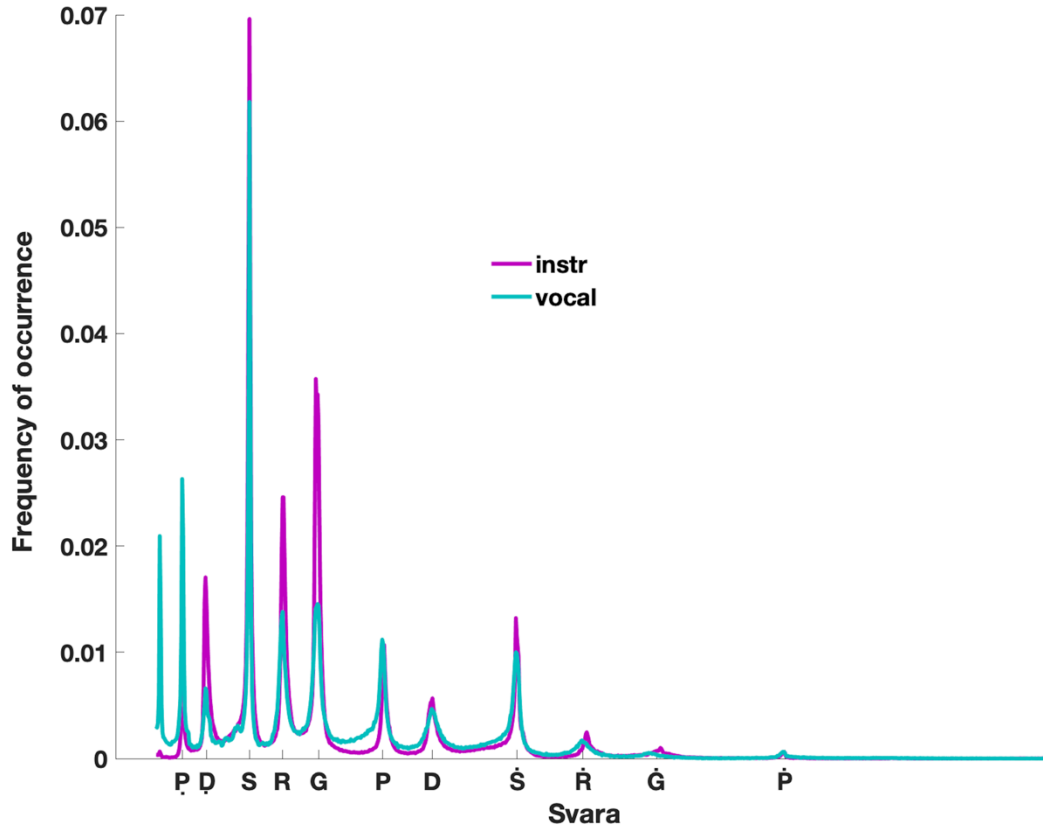


Figure 2. Probability distribution of the svaras for vocal (in turquoise) and instrumental (in purple) renditions of Bhūpālī. The high components of S and Ṗ could be due to the tānpurā, which provides the drone in the background.

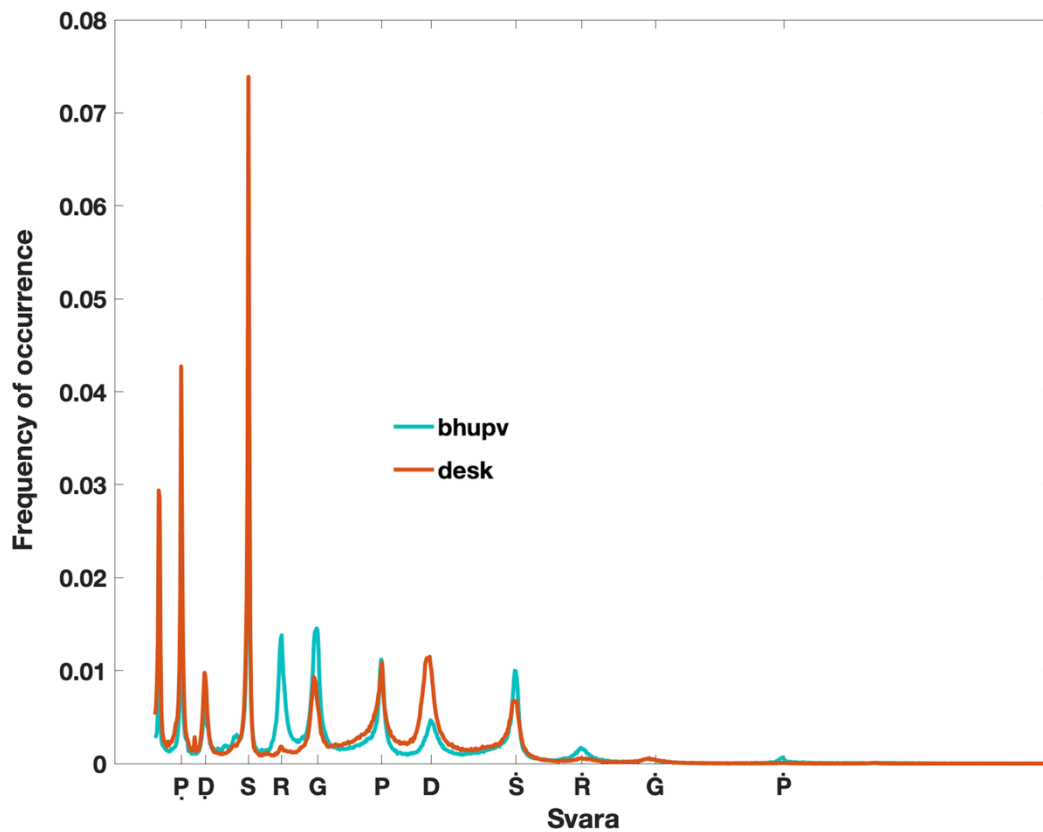


Figure 3. Probability distribution of the svaras of Bhūpālī (vocal only, in turquoise) and Deskār (in orange). The high components of S and Ṗ could be due to the tānpurā, which provides the drone in the background.

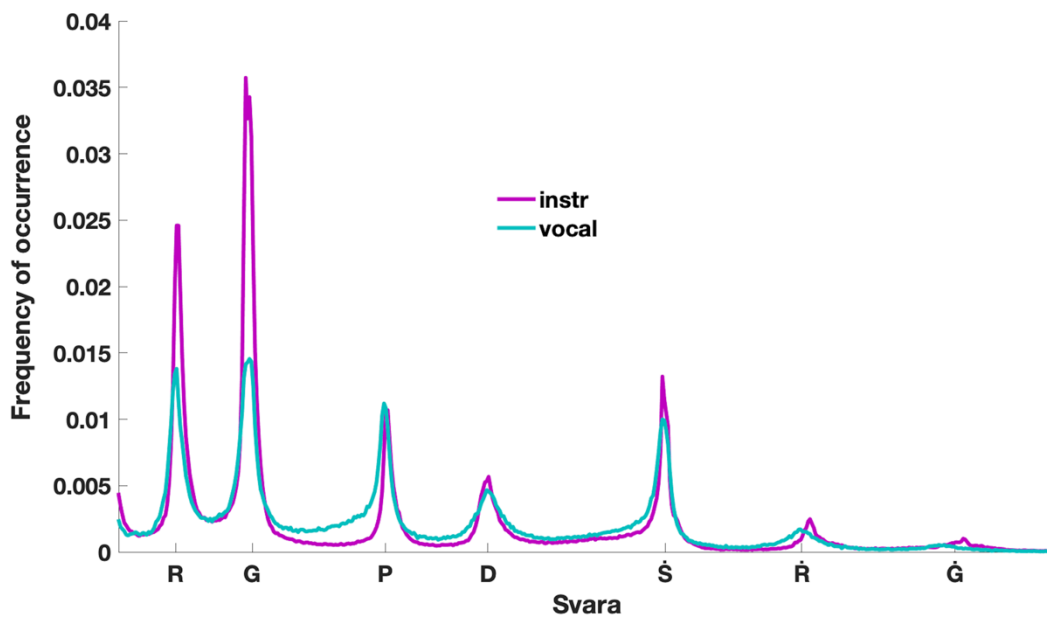


Figure 4. Zoomed-in view of the graph in Figure 2, showing the probability distribution only for *re, ga, pa, dha*, and *tāra sa, re and ga*. As before, the distribution for vocal recordings is in turquoise, and that for instrumental renditions is in purple.

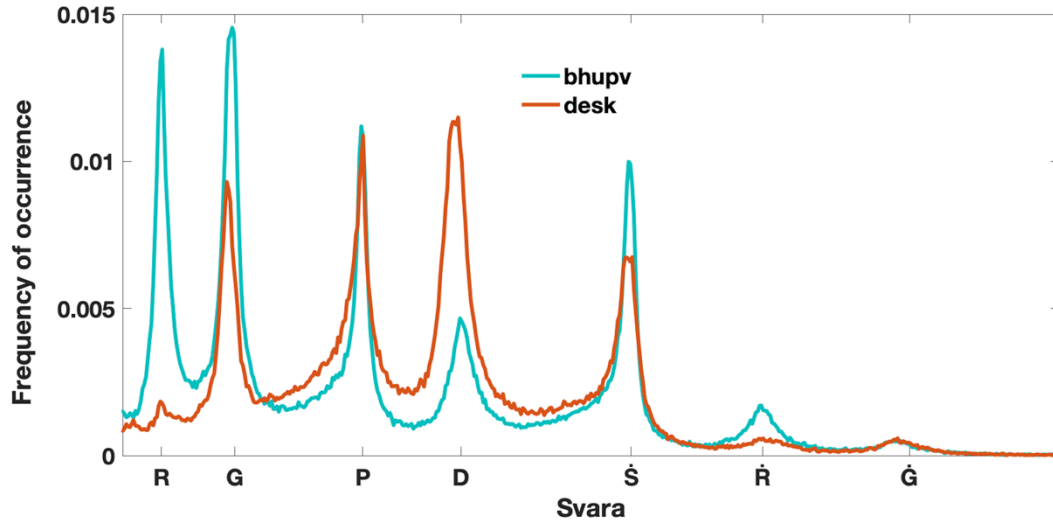


Figure 5. Zoomed-in view of the graph in Figure 3, showing the probability distribution only for *re*, *ga*, *pa*, *dha*, and tāra *sa*, *re* and *ga*. As before, the distribution for Bhūpālī (vocal only) is in turquoise, and that for Deskār is in orange.

2. ALPATVA AND BAHUTVA OF RE, GA, PA AND DHA IN TERMS OF THE PROBABILITY DISTRIBUTION

[41] In this part, only *re*, *ga*, *pa* and *dha* are compared. The sound of the tānpurā possibly increases the component of mantra *pa* and madhya *sa*. Therefore, I cannot conclusively comment on the alpatva or bahutva of these svaras. The relative frequencies of these four svaras are apparent in Figures 6 and 7.

[42] In Bhūpālī, *ga* is the svara that is used the most often. The second position goes to *re*, followed by *pa*. *Dha* tends to be somewhat understated. In contrast, in Deskār, the order is *dha*, *pa*, *ga*. *Re* is highly understated.

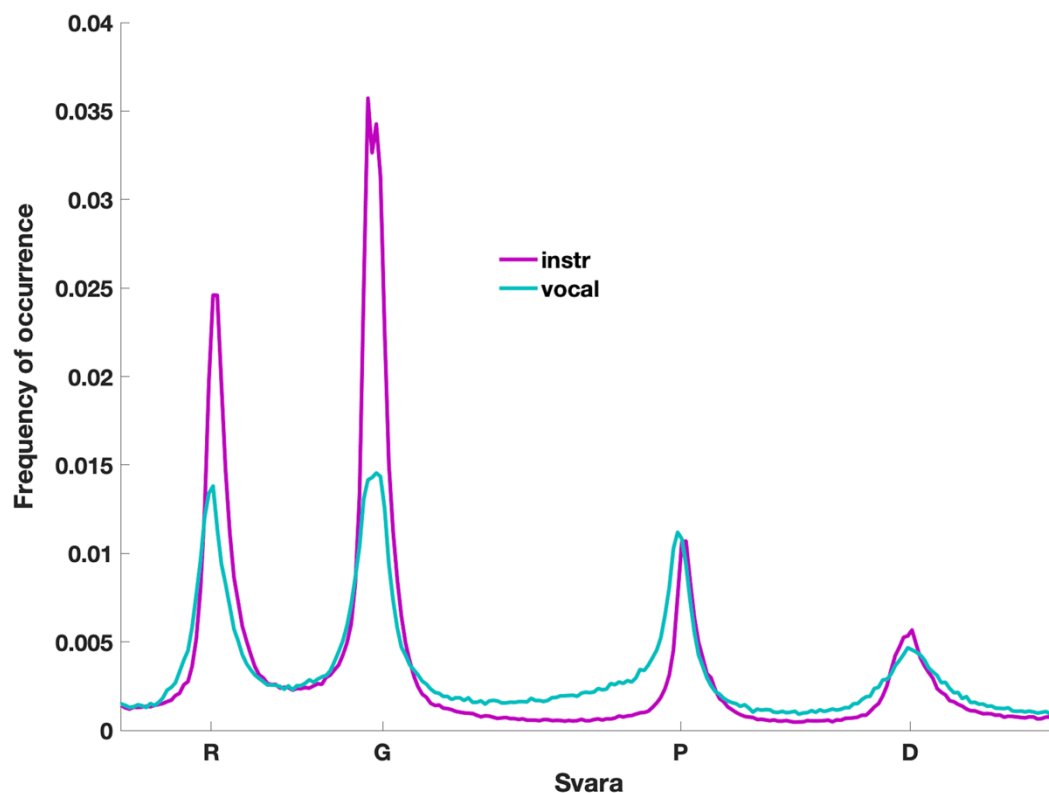


Figure 6. Zoomed-in view of the graph in Figure 2, showing the probability distribution only for *re, ga, pa* and *dha*. As before, the distribution for vocal recordings is in turquoise, and that for instrumental renditions is in purple.

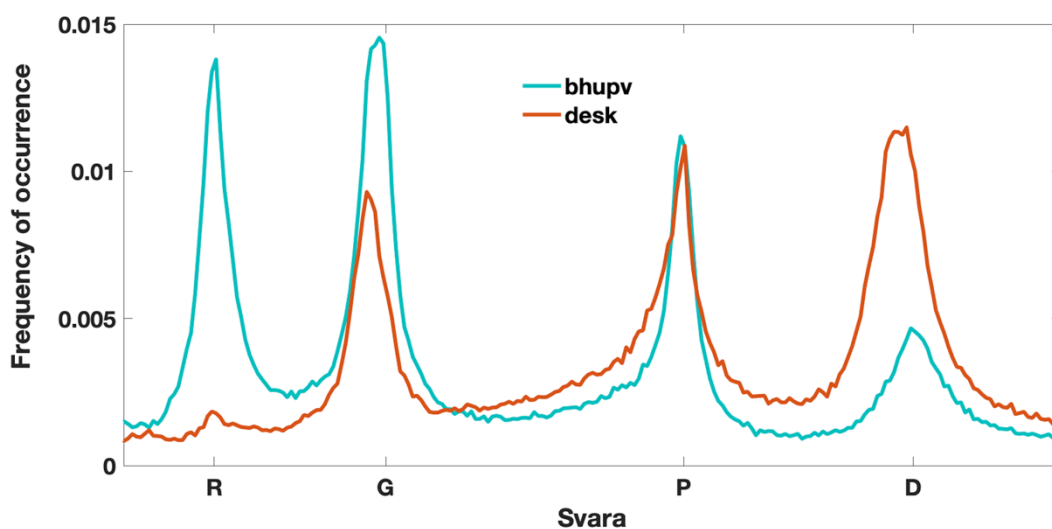


Figure 7. Zoomed-in view of the graph in Figure 5, showing the probability distribution only for *re, ga, pa* and *dha*. As before, the distribution for Bhūpālī (vocal) is in turquoise, and that for Deskār is in orange.

[43] I next measured two more parameters for each artiste—the D-ratio (Figure 8) and the G-ratio (Figure 9). These ratios respectively represent how often a given svāra occurs in comparison to how often *dha* (the vādī of Deskār but the samvādī of Bhūpālī) or *ga* (the vādī of Bhūpālī but the samvādī of Deskār) occurs. Thus, the D-ratio of a given svāra (*re*, *ga* or *pa*) is the ratio of the appearance frequency, $p(F_s)$, of this svāra (represented by its pitch frequency F_s) to the $p(F_s)$ of *dha*. The G-ratio of a svāra (*re*, *pa* or *dha*) is the ratio of the $p(F_s)$ of this svāra to the $p(F_s)$ of *ga*. The D-ratios and G-ratios for each recording of Bhūpālī and Deskār are shown in Tables 3 and 4 respectively. The means and standard deviations are shown in Table 5. The differences between Bhūpālī and Deskār in terms of alpatva and bahutva are clear upon inspection of these tables and of figures 8 and 9.

Bhūpālī						
Artiste	D-ratios			G-ratios		
	R/D ratio	G/D ratio	P/D ratio	R/G ratio	P/G ratio	D/G ratio
1. Vocal						
Kishori Amonkar	1.79	1.80	1.07	0.99	0.59	0.55
Kumar Gandharva	1.75	2.43	1.44	0.72	0.59	0.41
Prabhakar Karekar	1.98	2.16	1.58	0.91	0.73	0.46
Mashkoo Ali Khan	0.52	1.69	0.85	0.31	0.50	0.59
Rashid Khan	2.696	3.59	1.91	0.75	0.53	0.28
Padmini Rao	1.04	1.68	1.87	0.61	1.11	0.595
Padma Talwalkar	2.10	2.06	0.86	1.02	0.42	0.48
2. Instrumental						
Hariprasad Chaurasia	4.89	8.18	1.45	0.598	0.17	0.12
Rakesh Chaurasia 1	2.12	2.44	0.66	0.87	0.27	0.41
Rakesh Chaurasia 2	1.38	1.72	0.50	0.80	0.29	0.58
Rakesh Chaurasia 3	2.30	2.65	1.03	0.87	0.39	0.38
Rakesh Chaurasia 4	0.48	0.95	0.49	0.50	0.51	1.06
Pravin Godkhindi 1	1.36	1.15	1.00	1.18	0.87	0.87
Pravin Godkhindi 2	2.82	3.05	1.82	0.92	0.598	0.33
Bismillah Khan	3.74	8.295	2.19	0.45	0.26	0.12
Sultan Khan	3.08	5.395	1.60	0.57	0.297	0.18
Lalmani Misra	7.04	10.41	1.63	0.68	0.16	0.096
Gianni Ricchizzi	1.13	3.18	0.65	0.36	0.20	0.32

Table 3. Ratios of appearance frequency: R to D, G to D, P to D (D-ratios) and R to G, P to D, D to G (G-ratios), for Bhūpālī rendered by different artistes.

Deskār						
Artiste	D-ratios			G-ratios		
	R/D ratio	G/D ratio	P/D ratio	R/G ratio	P/G ratio	D/G ratio
Kishori Amonkar	0.11	0.72	0.48	0.15	0.67	1.39
Arati Ankalikar-Tikekar	0.13	0.46	0.54	0.29	1.18	2.17
Ajoy Chakraborty	0.19	0.58	1.13	0.32	1.95	1.72
Kumar Gandharva	0.12	0.55	1.15	0.22	2.11	1.83
Bhimsen Joshi	0.15	0.65	1.05	0.23	1.61	1.53
Archana Kanhere	0.03	0.15	0.60	0.22	4.12	6.87
Rashid Khan	0.17	0.60	0.73	0.28	1.22	1.67
Mallikarjun Mansur	0.11	0.36	0.61	0.30	1.71	2.80

Table 4. Ratios of appearance frequency: R to D, G to D, P to D (D-ratios) and R to G, P to G, D to G (G-ratios), for Deskār rendered by different artistes.

Ratio		Bhūpālī vocal		Bhūpālī instrumental		Bhūpālī all		Deskār	
		mean	SD	mean	SD	mean	SD	mean	SD
D-ratios	R/D	1.7	0.66	2.76	1.82	2.34	1.57	0.13	0.04
	G/D	2.2	0.62	4.31	3.11	3.49	2.67	0.51	0.17
	P/D	1.37	0.42	1.18	0.56	1.26	0.52	0.79	0.26
G-ratios	R/G	0.76	0.23	0.71	0.23	0.73	0.23	0.25	0.05
	P/G	0.64	0.22	0.36	0.21	0.47	0.25	1.82	0.97
	D/G	0.48	0.10	0.41	0.30	0.43	0.25	2.5	1.70

Table 5. Mean values and standard deviations for the D-ratios and G-ratios for both rāgas. Shaded blocks indicate the vādī to saṁvādī ratios of each rāga.

[44] I shall first discuss the vādī to saṁvādī ratios (shaded in green)—*ga* to *dha* in Bhūpālī, and *dha* to *ga* in Deskār. The śāstras indicate that the vādī should have greater bahutva than the saṁvādī, but do not give quantitative information. Therefore, all we know from the śāstra is that this number should be greater than 1. The present study reveals more information. The vādī to saṁvādī ratios (G/D) for the Bhūpālī recordings in this study range from 0.95 to 3.6, with the mean being 3.49 and standard deviation being 2.67—except for four outliers, which I will discuss subsequently. The corresponding ratios for

the Deskār recordings—i.e., the D/G ratios, since the vādī of Deskār is D and the saṁvādī is G—range from 1.5 to 2.8, except in Archana Kanhere’s recording, where the ratio approaches 7. The mean value is 2.5, and the standard deviation is 1.7.

[45] As for the *re* (the alpa svara) of Deskār, its mean D-ratio is 0.13, and mean G-ratio is 0.25. Both values are far less than 1. However, the *re* of Bhūpālī has a mean D-ratio of 2.34. Its mean G-ratio is 0.73. This suggests that *re* and *ga* are often used with roughly similar frequencies in this rāga. Interestingly, in one case—the recording of Pravin Godkhindi—the G-ratio of *re* (1.36) indicates that it has been used to an extent slightly greater than the vādī *ga*.

[46] The instrumental Bhūpālī recordings have higher mean D-ratios of R and G as compared to the vocal ones. The main contribution to this comes from the recordings of Lalmani Misra (vicitra vīṇā), Hariprasad Chaurasia (flute), Bismillah Khan (śahnā’ī) and Sultan Khan (sāraṅgī). These “outliers” as seen in Figure 8 seem to indicate that the approach of instrumentalists to a rāga may differ from that of vocalists. This is however too small a sample to make a conclusive claim. Not only does each individual musician have a different approach, but the same musician also renders the same rāga differently on different occasions.

[47] The mean D-ratio of the *pa* of Deskār is 0.79 (SD = 0.26), suggesting that *pa* and *dha* are used nearly to the same extent in this rāga. In three cases, *pa* appears to have been used to an extent slightly greater than the vādī *dha*. As for the G-ratios of *pa*, they lie roughly between 1.6 and 4, with the mean being 1.82. Thus, in all cases, *pa* has greater bahutva than *ga* in Deskār.

[48] I now move to the ratios of *pa* in Bhūpālī. The D-ratios and G-ratios show a large spread—0.5 to 2.8 in the former case, and 0.2 to 1.3 in the latter case. The mean values are 1.26 (SD = 0.52) and 0.47 (SD = 0.25) respectively. These results are shown pictorially in Figures 8 (D-ratios) and 9 (G-ratios), where each point represents a particular artiste. The distribution of D-ratios of Bhūpālī has no overlap with the corresponding distribution for Deskār. Similarly, there is a clear divide between the G-ratio distributions. Can such ratios help in raga recognition? To answer this question, I performed a k-means analysis (explained below) on the G-ratios and the D-ratios, to divide the data into two clusters. In the case of the G-ratio, the clusters identified were largely identical to Bhūpālī and Deskār. However, in the case of the D-ratio, the four outliers were identified as one cluster and all other data points were grouped into another cluster. The outliers are all instrumentalists. I had earlier commented upon differences in their renditions of ragas

compared to vocalists. Thus, in the case of Bhūpālī and Deskār, k-means analysis may be said to be capable of distinguishing between them if only vocal recordings are used. This study does not include instrumental recordings of Deskār.

K-MEANS ANALYSIS

[49] To categorize the G-ratios and D-ratios, I used k-means analysis. This is a widely-used clustering technique used to classify datapoints into two or more distinct clusters. The “k” in the name of the method refers to the number of mean values or centroids. In the case of the present work, $k = 2$. After the number of means has been defined, one feeds in guess values for each mean. The algorithm groups the datapoints based on which mean they are the closest to. After this, it calculates means for each cluster, uses these as the guess means, and repeats the steps. The process is iterated until there is no change in the means anymore. I used this technique to attempt to classify the recordings as Bhūpālī or as Deskār. However, the results obtained were different.

[50] k-means analysis on the D-ratios and G-ratios correlated well with figures 8 and 9. When the analysis was done on the D-ratios, all renditions of Bhūpālī were grouped along with the Deskār recordings, except for the four instrumental outliers in Figure 8: Lalmani Misra, Hariprasad Chaurasia, Bismillah Khan and Sultan Khan. The same result was obtained when the analysis was carried out only on the Bhūpālī recordings. When the analysis was repeated after leaving out the instrumental recordings, Mashkoor Ali Khan’s rendition (Bhūpālī vocal) was grouped along with the Deskār recordings.

[51] When the analysis was done on the G-ratios, Kishori Amonkar’s vocal recording of Bhūpālī was grouped along with the Deskār recordings. This could be owing to her well-known unconventional approach to rāgas. The same result was obtained when k-means analysis was carried out after leaving out the instrumental recordings. However, in the plot, the data point for Kishori Amonkar’s Bhūpālī appears separate from the Deskār cluster.

[52] The k-means analysis could not effectively separate the vocal and instrumental recordings of Bhūpālī in terms of G-ratios.

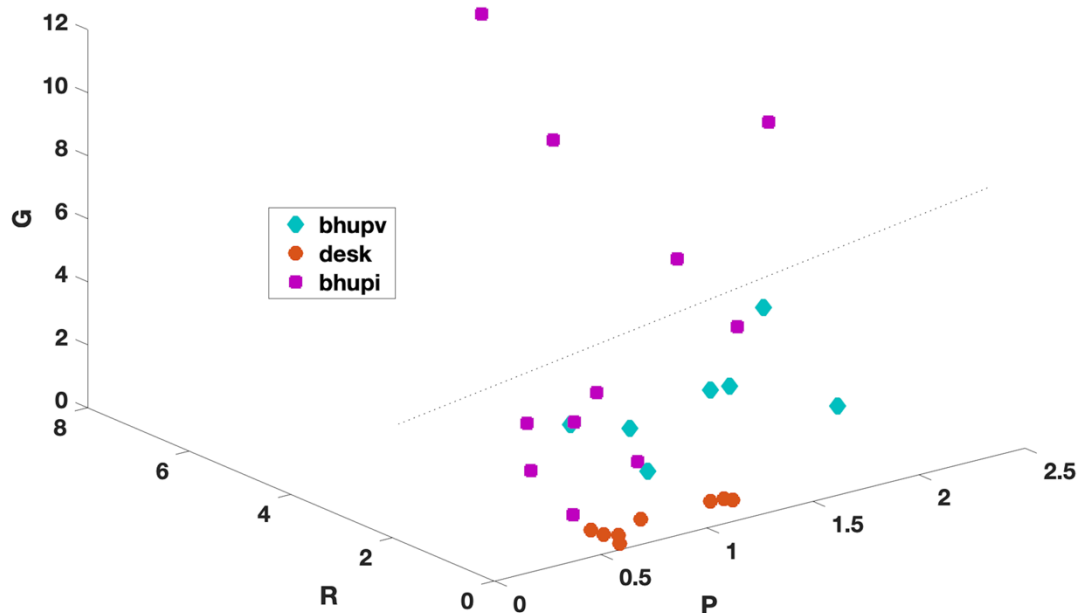


Figure 8. D-ratios: Ratios of frequency of occurrence of *re*, *ga* and *pa* relative to *dha*, for Bhūpālī and Deskār, all artistes. Turquoise diamonds: D-ratios for Bhūpālī (vocal), magenta squares: D-ratios for Bhūpālī (instrumental), orange dots: D-ratios for Deskār. The four outliers in this plot (separated by the dotted line for easy viewing) represent the instrumental recordings of Bhūpālī by Lalmani Misra, Hariprasad Chaurasia, Bismillah Khan and Sultan Khan.

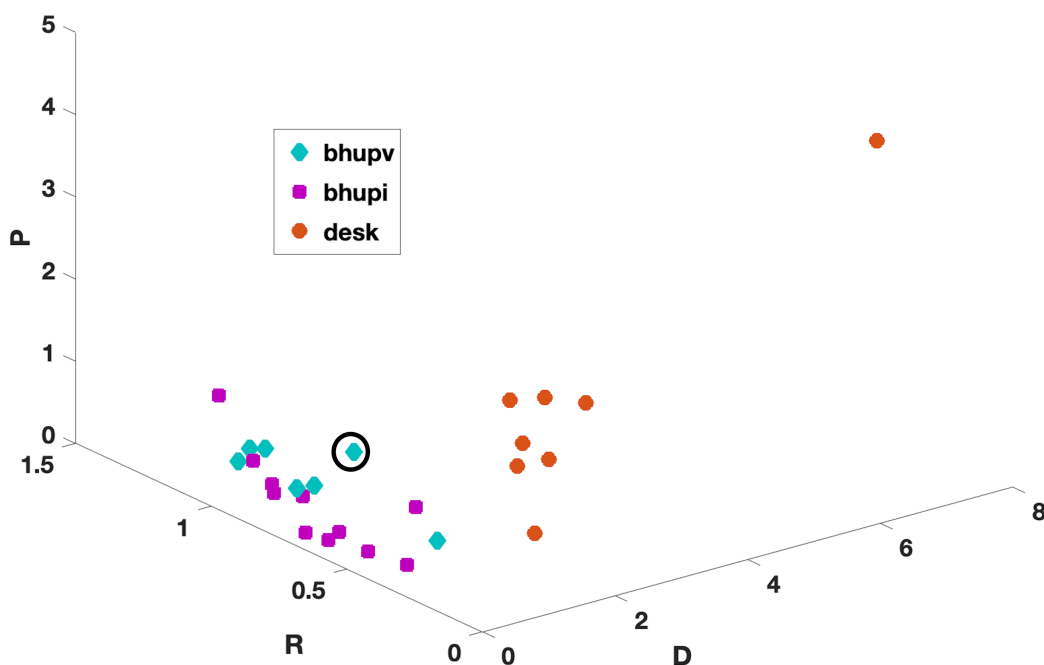


Figure 9. G-ratios: Ratios of frequency of occurrence of *re*, *pa* and *dha* relative to *ga*, for Bhūpālī and Deskār, all artistes. Turquoise diamonds: G-ratios for Bhūpālī (vocal), magenta squares: G-ratios for Bhūpālī (instrumental), orange dots: G-ratios for Deskār. The datapoint representing Kishori Amonkar, which gets grouped along with the Deskār recordings in the k-means analysis, is encircled.

SVARA-SAṄGATIS

[53] Transition matrices for svara pairs can be obtained by calculating their frequency of appearance. The transition matrices for Bhūpālī for all artistes are shown in Figure 10 (vocal) and Figure 11 (instrumental), and for Deskār in Figure 12. The y -axis represents the first svara of the pair, while the x -axis represents the second.

[54] The transition matrices show a near-tridiagonal structure, especially for Bhūpālī. The main diagonal consists of zeroes, since SS, RR, GG, PP etc. are not svara-saṅgatis. The first diagonals above and below the main diagonal indicate that pairs of consecutive svaras (SR, RS, RG, GR, GP, PG, PD, DP, DŚ, ŚD etc.) are more common than other pairs (SG, GS, RP, SP, PS, SD, ḌR, ḌG etc.). Such matrices may be made for svara pairs where each svara is held constant for a given time interval. Here the time interval is taken to be 20 milliseconds.

[55] In Bhūpālī, RG and GR appear much more often than other pairs, and RG appears more than GR. PD and DP are the next most common pairs, with PD appearing more frequently than DP. The high values for SR and RS for the Bhūpālī recordings could be tānpurā-influenced, and therefore, I have limited this part of study to the saṅgatis of R, G, P and D.

[56] In Deskār, the highest position is taken by DP and PD, followed by ŚD, DŚ, PG and GP. The calan of Deskār is highly uttarāṅg-dominated, and more compact than that of Bhūpālī.

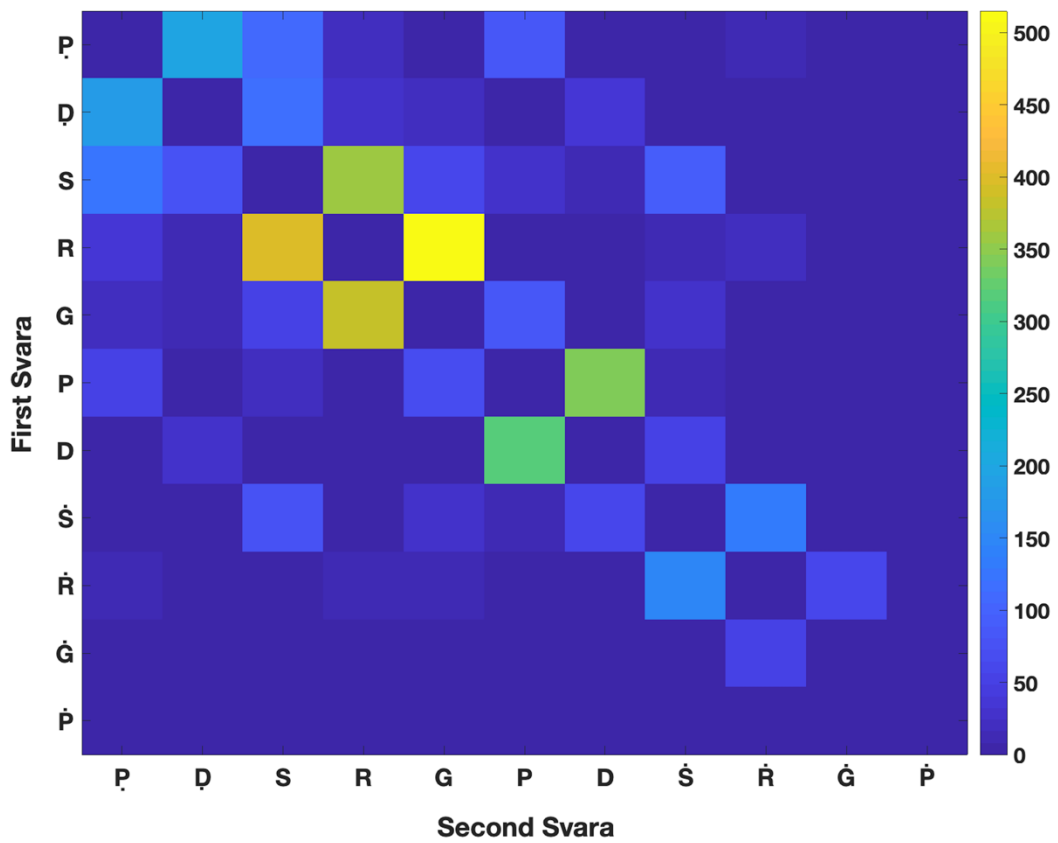


Figure 10. Transition matrix for Bhūpālī (vocal), all artistes. Each block represents a svara-saṅgati (pair of svaras that come one after another). The y-axis represents the first svara of the saṅgati, and the x-axis represents the second svara.

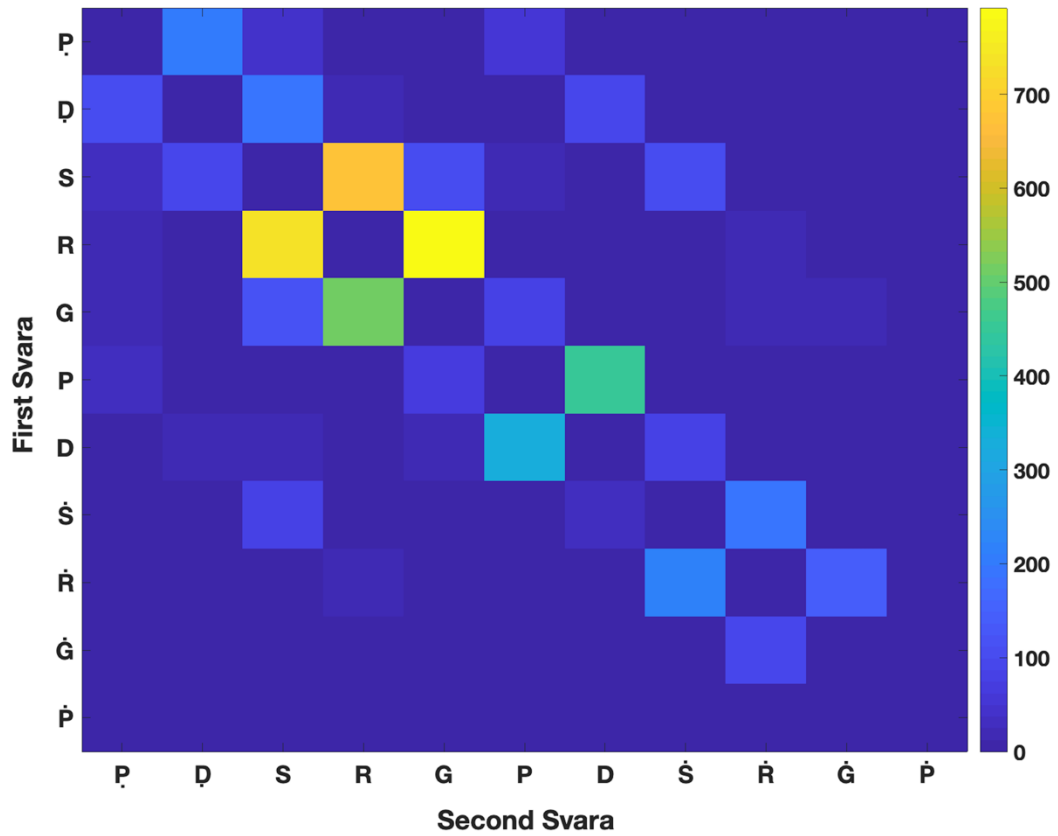


Figure 11. Transition matrix for Bhūpālī (instrumental), all artistes. Each block represents a svara-saṅgati (pair of svaras that come one after another). The y-axis represents the first svara of the saṅgati, and the x-axis represents the second svara.

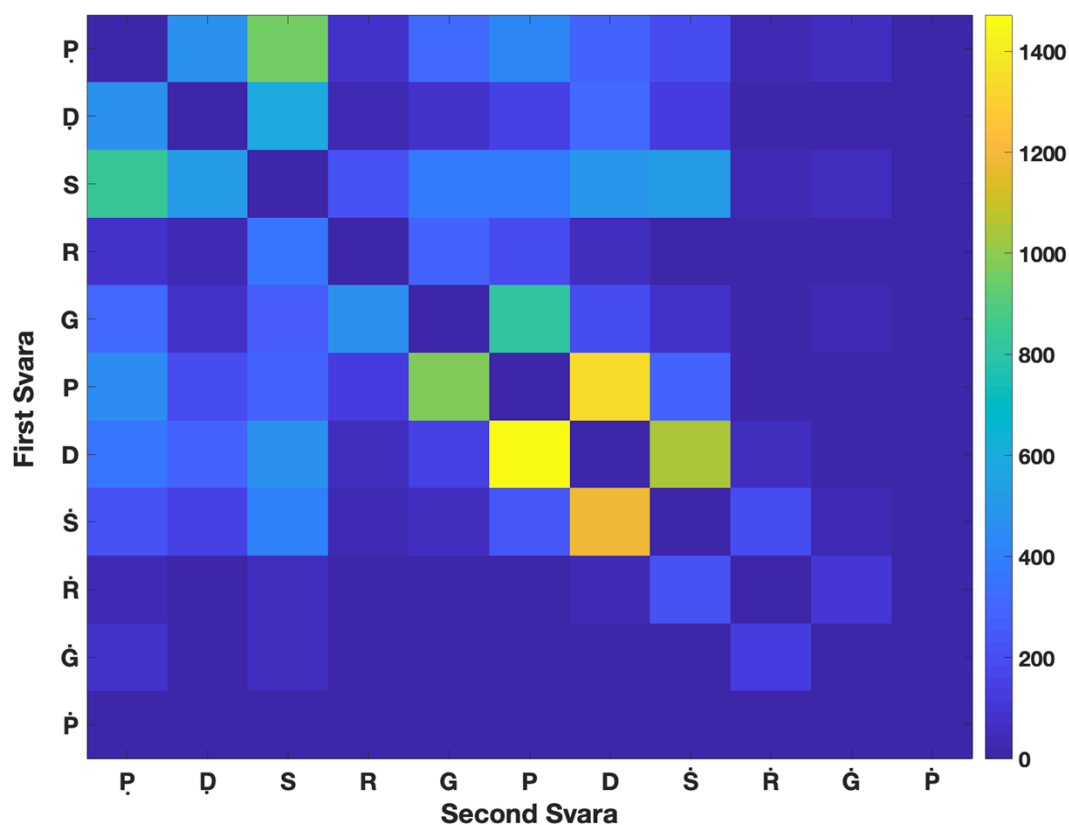


Figure 12. Transition matrix for Deskār, all artistes. Each block represents a svāra-saṅgati (pair of svaras that come one after another). The blocks for SP and ṖS represent the tānpurā drone, and have nowhere been taken into consideration. The y-axis represents the first svāra of the saṅgati, and the x-axis represents the second svāra.

[57] It is interesting that in both rāgas, the strongest svāra-saṅgatis involve the vādī (G for Bhūpālī and D for Deskār) and the svāra immediately below it. I term these *Type 1 vādī saṅgatis*. These saṅgatis are RG and GR in Bhūpālī, and PD and DP in Deskār. In Bhūpālī, the corresponding saṅgatis for the saṁvādī also have relatively high values.

[58] Let us now look at the saṅgatis involving the vādī and the svāra immediately above it, which I shall call *Type 2 vādī saṅgatis*. These are GP and PG in Bhūpālī, and DŚ and ŚD in Deskār. The Type 2 saṅgatis in Deskār (DŚ and ŚD) are second in frequency to the Type 1 saṅgatis (PD and DP). However, in Bhūpālī, the saṅgatis that are second in frequency to the Type 1 vādī saṅgatis (RG and GR) are SR and RS, which do not involve the vādī at all. This brings out a difference in the role of the vādī in the two rāgas.

[59] The transition matrix of Deskār indicates that among the most frequent pairs, the avarohī (descending) pairs are more frequent than the corresponding ārohī (ascending)

pairs. In other words, DP is more frequent than PD, PG is more frequent than GP, and so on. The difference is small, but consistent. The opposite pattern is observed in Bhūpālī. This is in accordance with Bhatkhande's observation that uttarāṅg-pradhān rāgas (such as Deskār) place greater importance on the avaroh than the āroh, while pūrvāṅg-pradhān rāgas (like Bhūpālī) are āroh-dominated (Bhatkhande 1920s–30s vol.5, 34).

[60] The matrix of Deskār does not give conclusive information about the laṅghan (svara skipping) of *re* in the āroh (represented by the saṅgati SG), because saṅgatis involving svaras lower than *ga* appear to be much less common than saṅgatis from GP/PG and upwards.

[61] The question arises whether the near-tridiagonal structure of the matrices would still hold for a highly vakra rāga (one with many randomly ordered characteristic phrases) like Gauḍ Sāraṅg, where zigzag phrases like GRmG, PDMP, SmGP, NDNMP and so on are rather common. Answering this question would require further study of rāgas of different *jāti* (categories in terms of number of svaras) such as auḍav (pentatonic), ṣaḍav (hexatonic) and sampūrṇa ('complete', or heptatonic, i.e., using all svaras of the *thāt*), or rāgas whose āroh and avaroh are of different *jātis*, and also extend this study to more vakra rāgas, to get an understanding of the variation in these transition matrices.

DĪRGHA BAHUTVA

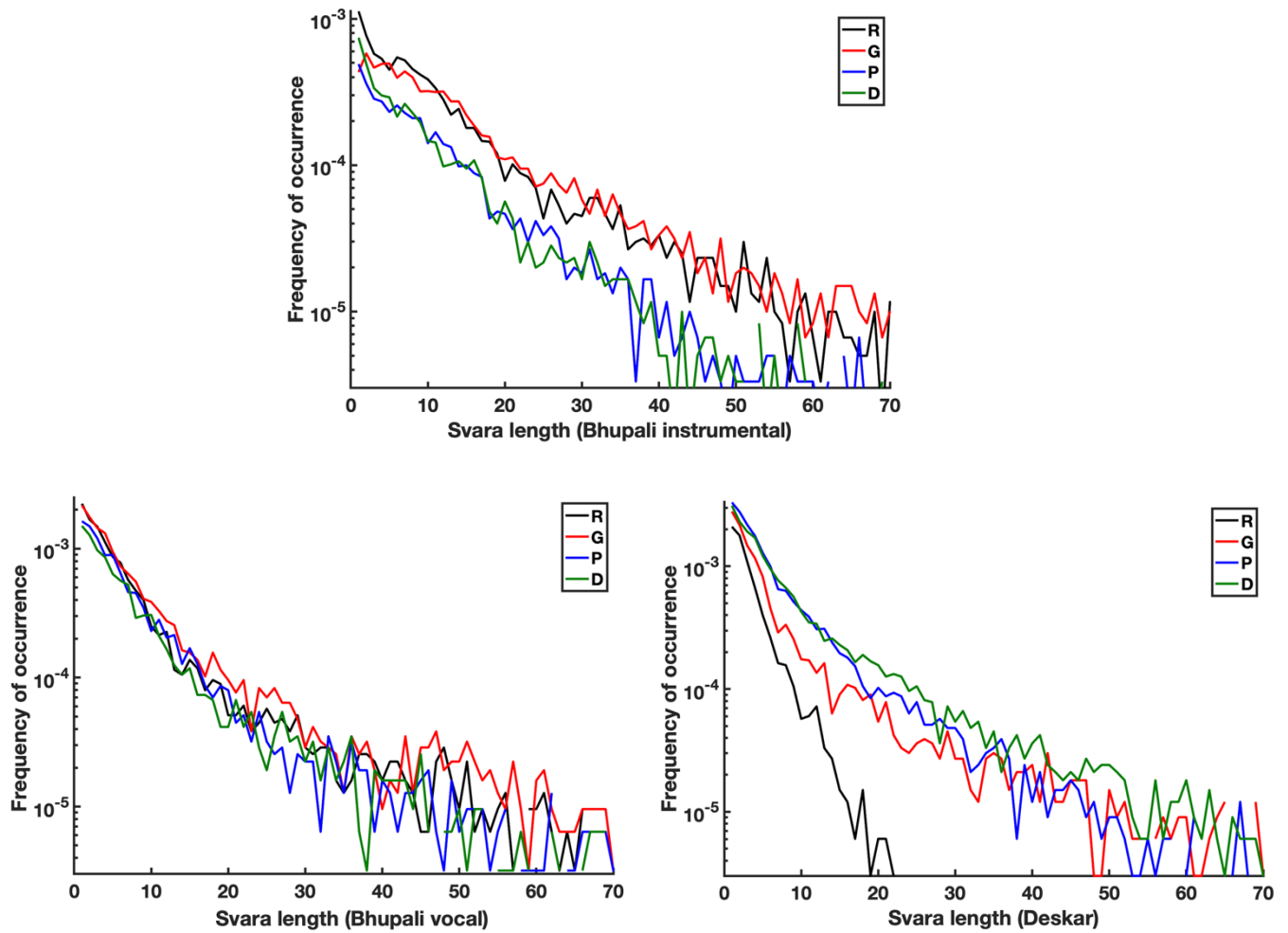


Figure 13. Length distribution of the svaras *re*, *ga*, *pa* and *dha* for Bhūpālī (instrumental and vocal) and Deskār, all artistes. The svara lengths are in centiseconds.

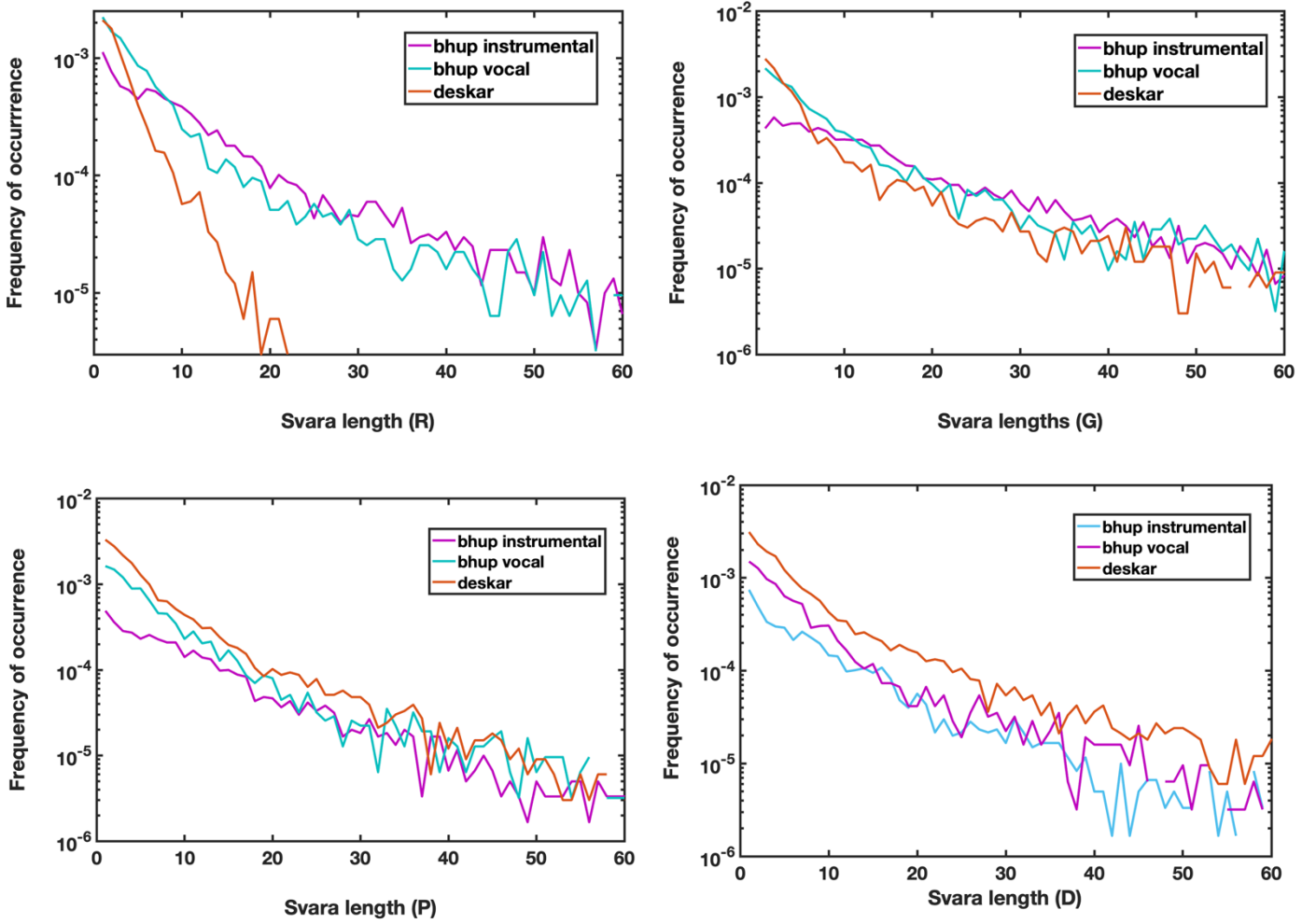


Figure 14. Length distribution of the individual svaras *re*, *ga*, *pa* and *dha* in Bhūpālī and Deskār, all artistes. The svara lengths are in centiseconds.

[62] The final strategy for quantitatively describing alpatva and bahutva used here is that of svara lengths, i.e., the length of time for which each svara is sung or played continuously. The data collected is summarized in Figures 13 and 14. In Deskār, *re* is highly reduced, and is rarely longer than 220ms. Long *pa* and long *dha* dominate, and appear more frequently, which can be seen in their graphs' close proximity and contour resemblance. This similarity indicates that in Deskār, both these svaras have abhyās bahutva to nearly the same extent. Their appearance frequency is greater than for long *ga* in Bhūpālī.

[63] In Bhūpālī, the svara with the greatest dīrgha bahutva (in terms of how often a svara is elongated) is *ga*, followed by *re*, *pa* and *dha*, in that order. This was confirmed by

computing average lengths.¹⁷ The differences in *dirgha bahutva* between any two of these *svaras* are smaller as compared to *Deskār*, where *pa* and *dha* have high *dirgha bahutva*, *re* is never *dirgha*, and *ga* is intermediate.¹⁸ The length distributions of *re* and *ga* in *Bhūpālī* instrumental recordings appear to have higher values than those of vocal recordings, owing to the four outliers discussed in the previous section.

CONCLUSIONS

[64] *Rāga* grammar is transmitted through the *śāstra*, which is both oral—passed on from guru to disciple—and textual. However, there are important characteristics of the *rāga* in the *lakṣya*, which are imbibed subconsciously by listening and imitating. This article, focused on the scale-identical *Hindustānī rāgas* *Bhūpālī* and *Deskār*, has been aimed primarily at bringing to light some of those characteristics which are not explicit in the *śāstra*, but are nevertheless consistently present across musicians and musical styles. This study also confirms and quantifies known features of these *rāgas*.

[65] My findings are summarized as follows:

- a) In terms of frequency of occurrence, as well as *dirgha bahutva* (elongation), it is common knowledge that the first position should go to the *vādī*. However, the *śāstra* does not define the relative positions of the other *svaras*. My work shows that there is a clear ranking. The *svaras* can be arranged in decreasing order as follows in *Bhūpālī*: G, R, P, D. The corresponding order for *Deskār* is D, P, G, R, with R very rarely being sung for longer than 220ms in the sample set, confirming R's place as *Deskār*'s *alpa svara*. Comparing the two *rāga* studied, the *alpatva* of D in *Bhūpālī* is less than the extreme *alpatva* of *re* in *Deskār*.
- b) I have devised a clustering method where I represent each performer by a point that represents the D-ratios (ratio of the frequency of occurrence of a *svara* to that of D) of R, G and P, or the G-ratios (ratio of the frequency of occurrence of a *svara* to that of G) of R, P and D, for that particular performance. I chose these particular *svaras* for the following reason: G is the dominant *svara* (*vādī*) of *Bhūpālī* and the *saṁvādī* (*svara* in agreement with the *vādī*) of *Deskār*. As for D, it is the *vādī* of *Deskār* and the *saṁvādī* of *Bhūpālī*. Measurements of the G-ratios and D-ratios for all recordings yielded 3D plots where the distributions of *Bhūpālī* recordings were clearly separated from the corresponding distributions of *Deskār*. Four

17. The average lengths for *Bhūpālī* (in centiseconds) are – G: 11.47, R: 9.81, P: 8.93 and D: 8.48.

18. The average lengths for *Deskār* (in centiseconds) are – D: 8.19, P: 6.72, G: 6.60 and R: 3.27.

instrumental Bhūpālī recordings—those of Lalmani Misra (vicitra vīṇā) and Hariprasad Chaurasia (flute), Bismillah Khan (śahnā'ī) and Sultan Khan (sāraṅgī)—are clearly separated from all other recordings of Bhūpālī, in terms of their unusually high D-ratios for R and G. This indicates a difference between the approach to Bhūpālī of instrumentalists and that of vocalists and suggests a possible method for computer-assisted rāga identification. These ratios are to be studied in greater detail and taking into account aspects other than alpatva and bahutva would be an interesting future direction to pursue.

- c) The śāstra says that the vādī is dominant but does not say by how much. My work quantifies this. The plot for *re* of Deskār is at a large distance away from the other plots. The plots show that G of Deskār has dīrgha bahutva that is comparable to that of R, P and D in Bhūpālī.
- d) As expected for non-vakra rāgas, svāra-saṅgatis (svāra pairs) of consecutive svaras are much more common than saṅgatis of svaras that are not immediate neighbours. One may anticipate that pairs that include the vādī would have the highest occurrence. These pairs in Bhūpālī are RG, GR, GP and PG. In Deskār, they are PD, DP, DŚ and ŚD. My work shows a hierarchy among these pairs as well. RG and GR have far higher values than GP and PG in Bhūpālī. In fact, PD and DP—the saṅgatis involving the saṁvādī—have higher values than GP and PG, and are second to RG and GR. In Deskār, on the other hand, the first position goes to PD and DP, and DŚ and ŚD are second to these. The saṅgatis involving the saṁvādī come third. These findings point to a difference in the function of the vādī between the two rāgas.
- e) The svāra one scale degree below the vādī (R in Bhūpālī and P in Deskār) appears to be nearly as strong as the vādī itself. The vādī is considered the 'king', the saṁvādī the 'minister', and the anuvādīs the 'servants' (Bhatkhande 1920s–30s vol.2, 14–15). However, in Bhūpālī and Deskār, anuvādīs have been shown to have greater bahutva than the saṁvādī. This indicates that rather than defining a single vādī and a saṁvādī svāra in agreement (in the sense of pūrvāṅg-uttarāṅg symmetry) with it, it would be more appropriate to define one or more *jīva svaras* ('lifeline' svaras) that govern the calan (overall structure) of the rāga. A *jīva svāra* is a svāra that is of special importance in a rāga. For example, it may be an important nyāsa¹⁹ svāra in the rāga, or it might be a svāra with a specific gamaka (i.e., alaṅkāra) by

19. The Sanskrit root word *nyāsa* (nom. sing. *nyāsaḥ*) is pronounced as *nyās* in Hindi.

which this particular rāga is distinguished from others. It may also be a point of high aesthetic value²⁰, or contributing to the unique identity of the rāga in other ways.²¹ While this is considered a Karnāṭik approach (e.g., Dīkṣitar 1904, 350), I see great utility for it in Hindustānī music too. Bhatkhande too alludes to this idea, without using the term “jīva svaras”. For example, he says that the beauty of the rāga Toḍī is dependent on the svaras r, g and d, while also mentioning that the vādī is d and saṁvādī is g. Similarly, in the case of Multānī, whose vādī-saṁvādī are *pa* and *sa*, he says that its aesthetic form is governed by g, P and N.²²

[66] Different artistes have different approaches to these rāgas in terms of which melodic motifs they use (as observed by listening to recordings), but an interesting interpretation of the data is that there is a broad similarity in the relative weightage that they give to different svaras (the G, R, P, D order for Bhūpālī, and D, P, G, R for Deskār), with some exceptions. I have confirmed this visually using individual probability distribution plots for each musician. Some examples are given in the appendix.

[67] In the future, it would be interesting to see whether similar numbers emerge for *re* in Śaṅkarā, a rāga where this svara appears to a comparable extent to its usage in Deskār. I envisage that the *ni* of Rāga Hiṇḍol would show even lower values than the *re* of Deskār or Śaṅkarā, since it is present as only a kaṇ in that rāga.

[68] There is of course a lot of inter-artiste variation, but there is also intra-artiste variation, in the sense that the same musician renders the same rāga differently on different occasions or on different instruments. The clustering method has the potential to quantify the uniqueness of styles of individual musicians, and also the uniqueness of each rendition of the same rāga by a given musician.

[69] In the theory of Hindustānī music, the svara known as vādī is defined as the svara with the greatest bahutva. The śāstra—for example, Bhatkhande’s *Kramik Pustak Mālikā*—

20. *Raṅjana*, described in Footnote 2.

21. To give some Karnāṭik examples, as described in Subbarāma Dīkṣitar’s 1904 text the Sangita Sampradāya Pradarśinī: For the rāga Śrī, the jīva and nyāsa svara is R. For Kalyāṇī, there are two jīva and nyāsa svaras – G and R. Mohanam, the Karnāṭik rāga closest to Bhūpālī and Deskār, has three – G, D and R. I assume that Dīkṣitar’s arrangement of the svaras in this order qualitatively indicates a decreasing order of bahutva (Dīkṣitar 1904 pp. 461, 1159, 1186).

22. Both these rāgas belong to the Toḍī thāṭ (SrgMPdN). But the difference in the vādī-saṁvādī and “jīva svaras”, along with the difference in āroh-avaroh (SrgMPdNS/ SrgMdNS – SNdPMgrS for Toḍī, and ṆSgMPNS – SNdPMgrS for Multānī) and uccāraṇ of the svara *ga* (ṛg in Toḍī, Ṁg in Multānī) makes all the difference between them.

says that *ga* is the vādī of Bhūpālī, and *dha* of Deskār. However, my study of the svara-saṅgatis and relative dominances of svaras reveals that the svaras immediately previous to them, *re* and *pa* respectively, are in practice often given nearly the same importance as the vādī. In some renditions, their bahutva might even be slightly higher than the bahutva of the vādī (for example, higher occurrence frequency of R as compared to G in Pravin Godkhindi's Bhūpālī recording, or higher occurrence frequency of P as compared to D in Kumar Gandharva's Deskār recording). Personally, I have long felt that rather than only stressing upon which svaras should be the vādī and saṁvādī, it makes more sense to say that the rāga has a number of *jīva svaras* ('lifeline' svaras) and *nyās svaras* (svaras that are resting points), and all these svaras contribute to its calan. The *jīva svara* approach is more common in Karnāṭik music, but the present study indicates that it holds good in Hindustānī music too. The vādī has been called the 'king' in some musical texts. Apart from having the greatest bahutva, it might 'rule' the calan of a rāga in other, more subtle ways. The subtleties revealed by the present study suggest that current rāga theories can be refined by studying a larger number of rāgas with methods similar to those used here. Also, it would be of interest to do similar studies of abundance and scarcity of musical notes in Karnāṭik music, and also in Western modes, Persian dastgāh, Arabian maqāms, Turkish and Balkan makams, and in similar melodic systems from the Far East or South-East Asia. Comparison of these other systems to Indian rāga systems could yield results that are hitherto unknown.

[70] Also, the Hindustānī *tāla* (musical meter) system has some beats that are emphasized more than others. Another question to ask would be which svaras more commonly occur on which type of beats, similar to the statistical methods used by Clayton (2020).

[71] Not only does the approach to one rāga vary from one person's rendition to another, it also varies within one person's lifetime. Each time the rāga is sung or played, it shows a different form of itself to the musician and the listeners. Using the methods established in this study, it would be an interesting exercise to quantify how rāga evolution takes place within the sphere of one musician's *riyāz* (practice) and performance.

[72] The present study can be extended using machine learning techniques, whose popularity in studies of Hindustānī music appears to be on the rise. There are an immense number of rāgas in Karnāṭik, Hindustānī and other such systems of music, and as discussed in the introduction, they have several features beyond alpatva and bahutva. Artificial intelligence (AI) could extract and classify data just by 'listening' to a large

corpus of recordings of several rāgas. It might thus be a crucial tool in developing a more refined general theory of rāgas. Another application of AI could be in distinguishing the actual music from the tānpurā drone.

[73] This study reminds us that the living music of rāgas is in the lakṣya, not the śāstra alone. Bhatkhande based his śāstra on the lakṣya of his time. As the lakṣya evolves, so must the śāstra. Studies such as the present one are important contributors to this process. It is interesting to study the evolution of rāgas from the medieval to the modern period using śāstras, but what is even more interesting to me is to study the evolution of the lakṣya on a much smaller timescale. Computational methods and AI could open up a wide range of possibilities in this regard.

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APPENDIX A

[75] Hindustānī music has seven *śuddha* ('pure', i.e. 'natural') svaras—S, R, G, m, P, D and N, corresponding to a major scale or Ionian mode in Western music. The other five—r, g, M, d, n—are known as *vikṛta* ('altered'). S and P are never vikṛta. Another way of grouping the svaras is as *komal* (flat) and *tīvra* (sharp), which applies to svaras other than S and P. The *śuddha* variants of *re*, *ga*, *dha* and *ni* are all *tīvra*. In the case of *ma*, the *komal* variant is considered *śuddha*.

[76] In the north, for a long time, the *śuddha* scale was SRgmPDn (Widdess 1995, 5). But in the 19th century, possibly owing to the introduction of the harmonium into Hindustānī music, the Major scale/ Ionian mode (SRGmPDN) was chosen as the *śuddha* scale, and this has continued into the present day (Jairazbhoy 1971, 22). Thus, R, m and D, which were earlier known as *śuddha ṛṣabha*, *śuddha madhyama* and *śuddha dhaivata*, are still known by the same names. The 'original' vikṛta svaras, G and N, which were earlier called *tīvra gāndhāra* and *tīvra niṣāda*, e.g., in the Saṅgīta Pārijāta of the 17th century, are now called *śuddha gāndhār* and *śuddha niṣād*, while g and n, which were

considered śuddha earlier, are now considered vikṛta, and called komal gāndhār and komal niṣād. Based on these twelve svaras, *mela*-s or parent scales are formulated (Powers et al. 2001; Bhatkhande 1910, 1920s–30s). A mela is a collection of seven svaras, *sa ri ga ma pa dha ni*, analogous to a heptatonic scale in Western music, examples of which are the major scale, natural minor scale and harmonic minor scale. In Hindustānī music, the term *thāt* is more commonly used instead of *mela*. The twelve svaras give rise to thirty-two thāts in Hindustānī music.

[77] *Komal* and *tīvra* are unambiguous terms. On the other hand, the definition of śuddha (natural) and vikṛta (altered) svaras varies with region and time. It is only the śuddha madhyam (m) whose definition has remained unchanged.

A BRIEF HISTORICAL BACKGROUND, AND ITS MODERN RELEVANCE

[78] Early (pre-16th-century) Indian music was based on seven musical notes, known as svaras. These were—and still are—named *śadja*, *ṛṣabha*, *gāndhāra*, *madhyama*, *pañcama*, *dhaivata* and *niṣāda*, and commonly known by their short names: *sa*, *ri*, *ga*, *ma*, *pa*, *dha*, *ni*. These notes were not defined relative to a system tonic as they are now. Rather, they corresponded to the Dorian mode beginning at the western note D (Widdess 1995 p.xv). In modern notation, these svaras would be S R g m P D n. Apart from the śuddha (‘natural’) notes, there were two additional notes, known as *vikṛta* (‘altered’) svaras. These corresponded to G and N. The svaras were partitioned between twenty-two microtonal intervals known as *śruti*-s.

[79] Śrutis (microtones) are still very important for Hindustānī and Karnāṭik music (S. Rao et al. ; S. Rao and van der Meer 2010). A śruti is a pitch, but not every śruti is a musical note (svara). Those śrutis that are actually used in a rāga or other melodic entity being performed are termed svaras (Bhatkhande 1920s–30s vol.4, 15). Though the twelve basic *svara-sthāna*-s (places of svaras) are as defined above, the same svara can vary in pitch depending on the rāga, being slightly higher (*caṛhā huā*) or lower (*utarā huā*) than its basic pitch, but still perceived and performed as that particular svara (S. Rao et al. ; S. Rao and van der Meer 2010). While some modern Hindustānī musicians base their svaras on the 22 śrutis (Bhatkhande 1920s–30s vol.4, 18), others—such as Chaitanya Kunte (Kunte 2017)—feel that in today’s time, there are not 22 but infinite śrutis, since the same svara has slightly different pitches in different rāgas, either by itself or with influence from other svaras. This work is concerned with comparing the relative abundance of the same

svaras in both rāgas, so the difference in exact pitches of the same svāra in both rāgas is not taken into account. However, I still felt it was important to discuss the śruti concept in brief here, since it is central to Hindustānī music. It would be interesting to incorporate this concept in future quantitative studies of rāgas.

[80] By the 16th century, *sa*, the tonic note, was no longer an absolute frequency. Instead, once it was defined as being at a particular pitch, the other notes would be defined *relative* to it.

[81] The precursor to rāga was known as jāti. It has the following *lakṣaṇas* (characteristics) (Qureshi et al. 2020; Widdess 1995, 47):

1. *Graha*: The note (svāra) that begins the melody (or a composition/song that has been composed in this melody)
2. *Aṁśa*: The svāra that is used the most frequently while singing the jāti. Widdess terms this the Predominant.
3. *Apanyāsa*: Svāras that act as end-points of phrases or of stanzas
4. *Nyāsa*: The svāra that ends the song
5. *Auḍava*: pentatonic nature
6. *Ṣāḍava*: hexatonic nature
7. *Tāra*: This refers to the higher octave. Here it means the limit in the higher octave above which the jāti cannot move.
8. *Mandra*: This refers to the lower octave. Here it means the limit in the lower octave below which the jāti cannot move.
9. *Alpatva*: translated by Widdess as “infrequent pitches”
10. *Bahutva*: translated by Widdess as “frequent pitches”

[82] The word *rāga* as a musical entity first appears in the 8th century (Qureshi et al. 2020). When jātis evolved into rāgas, most of the above lakṣaṇas continued to be used for them. These concepts have evolved over time. The musicologist Vishnu Narayan Bhatkhande (1860-1936) (Bhatkhande 1910, 193-, 1914, 1920s–30s), who is credited with having written the first modern treatise on Hindustānī music, carried out a detailed study of several ancient and medieval texts (Bhatkhande 193-) and received knowledge of south Indian music from the musician-scholar Subbarāma Dikṣitar (Bhatkhande 1904). Bhatkhande bases his rāga theory on a modified version of the theory in these sources, and the influence of his works persists to the present day, possibly to a greater extent than that of any other music scholar’s works.

[83] The term *jāti* has changed its meaning, and now refers to auḍav (pentatonic), śāḍav (hexatonic), sampūrṇa (heptatonic) and so on.

[84] The dominant svāra (*aṁśa*) is called *jīva svāra* (the svāra that is the lifeline) in modern Karnāṭik music, and *vādī* in modern Hindustānī music (Powers et al. 2001). Another svāra, known as the *saṁvādī*, is considered the second dominant svāra, and is in agreement (*saṁvād*) with the *vādī*. The concept of *saṁvādī* is a point of difference between modern Hindustānī music and the medieval music of northern India that preceded it. The *vādī* and *saṁvādī* of the modern Hindustānī rāga Bhūpālī are *ga* and *dha* respectively. Deskār, which shares the same scale, has *dha* for its *vādī* and *ga* for its *saṁvādī*.

[85] Nyās svaras, rather than being analogous to the word ‘finis’ used in the end of a book, are now the svaras on which phrases end, like ‘full stops’ of musical ‘sentences’. There is no one single nyās svāra on which the melody compulsorily must end. *Sa* is by definition a nyās svāra in every rāga, and there are one or more other svaras on which one may rest. For example, in Bhūpālī, *ga* is the other important nyās svāra apart from *sa*, and in Deskār, the nyās points are *dha* and *pa*.

[86] A major change happened in the 16th century or slightly earlier, with the introduction of more vikṛta svaras and the change in the definition of *sa* from a roughly absolute frequency to a “system tonic” relative to which the other svaras are defined (Qureshi et al. 2020). Music began to be defined in terms of twelve svaras: seven *śuddha* (‘pure’ or ‘natural’) svaras, and five vikṛta svaras.

APPENDIX B

A NOTE ON TĀNPURĀ REDUCTION

[87] A noise reduction study was conducted to estimate the effect of the tĀnpurā drone and other background noises. For this, the noise reduction feature on Audacity® 2.3.2²³ was used. A few seconds of the tĀnpurā-only segment in the beginning of the recording were selected as ‘noise’ and filtered out, as has been done in earlier studies (P. Rao, p.c. 2020; V.M. Rao 2011). Over a big majority of the signal, no effect of noise reduction was detected on visual inspection. But in small portions of the recording, spurious frequencies were removed and replaced by the signal, presumably made by the artiste. An example of the effect of noise reduction is shown in Figure B1. It is seen that noise reduction brings about a more continuous signal.

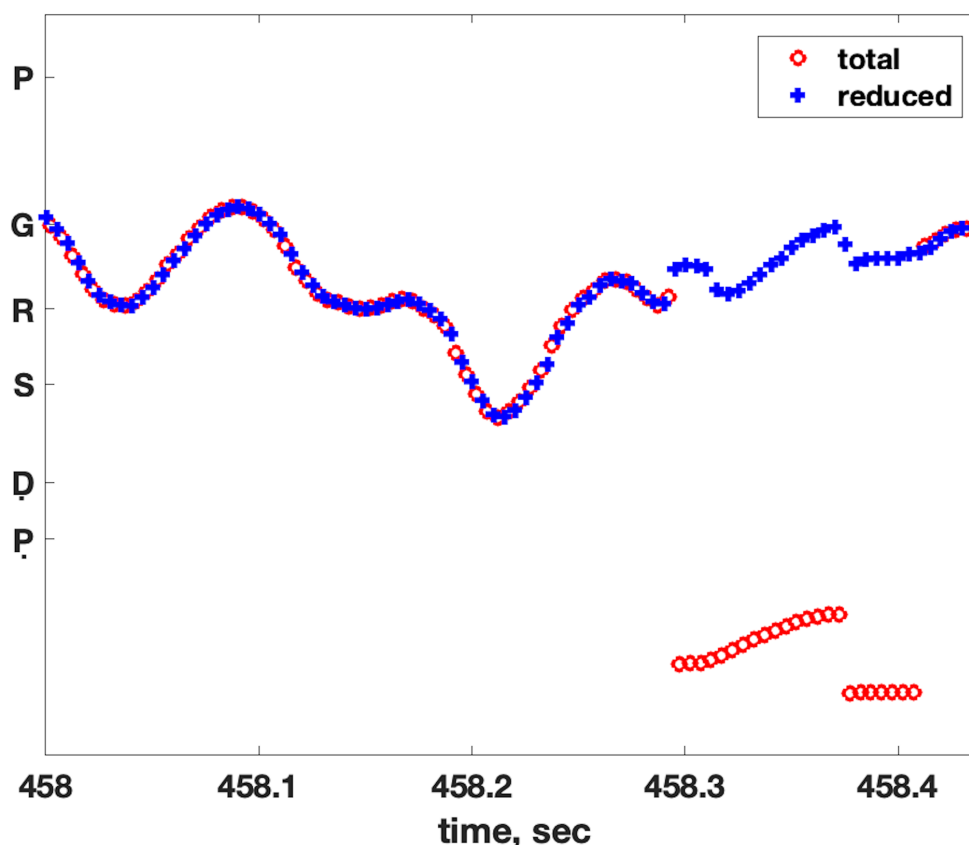


Figure B1. Noise reduction, a sample, from Rashid Khan’s Bhūpālī rendition. The red graph shows the raw data, and the blue graph shows the data after noise reduction. The svaras are indicated on the y-axis.

23. Version 2.3.2 retrieved from <https://audacityteam.org/> (2019). Audacity® software is copyright © 1999-2020 Audacity Team. Web site: <https://audacityteam.org/>. It is free software distributed under the terms of the GNU General Public License. The name Audacity® is a registered trademark of Dominic Mazzoni.

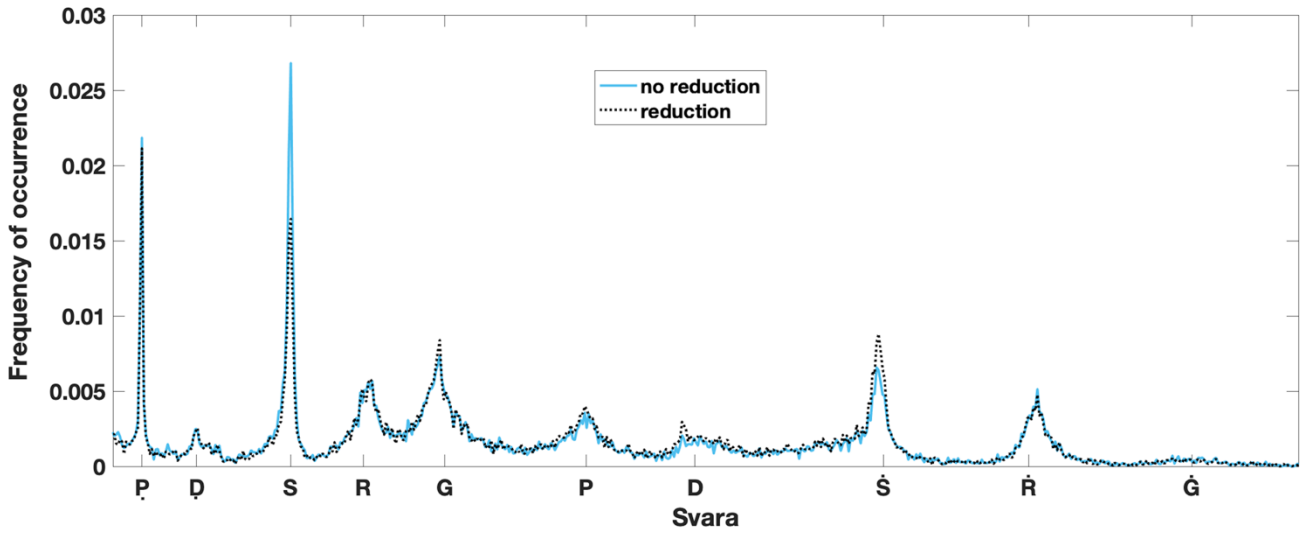


Figure B2. The probability density plot for Kumar Gandharva's Bhūpālī rendition, with and without tānpurā reduction (shown respectively by a blue solid line and black dotted line)

[88] Several of the recordings were analysed with and without noise reduction, and it was checked that, while the intensity of the madhya *sa* and the mandra *pa* were reduced, the ratios of the other svaras to each other were not significantly altered. Also, plots of different quantities measured in this study were made with and without noise reduction, and there was no qualitative difference. An example of this is shown in Figure B2.

[89] As can be seen in this figure, the reduction of S and Ḍ is not complete. A stronger reduction may lead to loss of intensity of the signal. I note that the reduction can be done using different parameters, and each time, there are small differences in the results. However, these are always minor for all the recordings, and do not alter any of the conclusions of this study. Overall, it was decided to follow the consistent approach of not using the reduction for any of the recordings analyzed in this study.

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